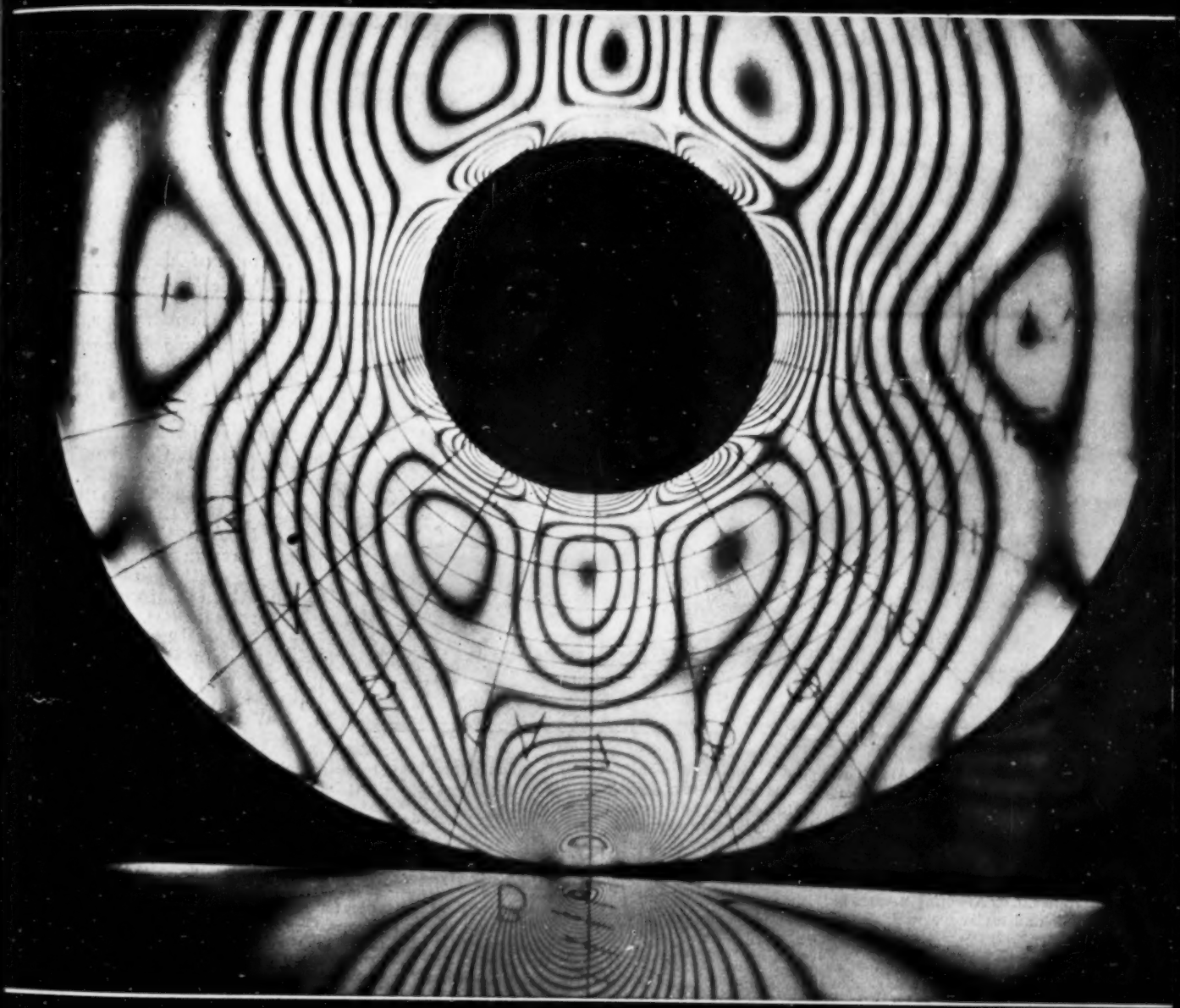
# JOURNAL OF APPLIED PHYSICS



JOURNAL OF APPLIED

# PHYSICS

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Volume 9 July, 1938 Number 7

# Scientific Training and Its Relation to Industrial Problems

BY C. F. KETTERING

*Vice President, General Motors Corporation, Detroit, Michigan*

I

WE are here to exchange our different points of view. Sometimes industrial people underestimate the importance of the advanced type of work which is done in universities. If we are trying to move an object with a block and tackle, we have to have a stake out ahead. The advance work carried on in universities is exactly like driving the stakes, well ahead of the load you are pulling. You can't pull it very far with each stake, and just the minute that you stop driving them, you can only pull the load up to where the last stake is. We must always have the new things way out ahead of the line, whether we can see any practical advantage or not.

A group of university presidents called on me about a year ago, and stated that the automobile industry has only a very few physicists in its organization. Why is it, they asked, that next to the coal industry, the automobile industry uses fewer technically trained men than any industry in the world. Other groups repeated that next to some other industries the automobile industry had fewer physicists. These statements came so close together that I thought there must be something to it. Although I was able to find some data later on which proved that these statements were not true, I was interested to find why they appeared to be true. Perhaps the automobile industry needs a different product from what the schools are turning out. I did

*For August*

*Some Physical Phenomena at the Temperature of Liquid Helium, by* E. F. BURTON

*Reflections on Rupture, by* P. W. BRIDGMAN

*Simple Electron Microscopes, by* R. P. JOHNSON

*Original research papers on the Characteristics of the Glow to Arc Transition in Mercury Vapor, X-Ray Investigation of the Form of Acetylsalicylic Acid in Certain Sugars, and Studies in Lubrication, Part II.*

* Paper presented at the Symposium on Physics in the Automotive Industry at Ann Arbor, Michigan, March 14 and 15, 1938.

know that we have thirty million cars running in this country, and if they didn't run, there would be something wrong with their physics.

I think that the people within the universities hardly appreciate the differences between a physicist inside an organization and one outside. A physicist within a university has only two factors to deal with, material and energy relationships. When you go into an industry you have four factors—material and energy relationship plus economics and psychology. I don't know whether the complications go up as the square or the cube of the factors involved when you add psychology and economics, but there is no question that they do increase. For instance, it isn't the fact that a machine will work that is important; it is whether the fellow for whom you make it likes it and is willing to pay for it. The latter is quite important, especially if you are going to make your living making machines. If you enter commercial research, you will find that sometime economics and psychology become more important than the pure functioning of the mechanical device itself.

## II

At our proving ground we have the up-school and the down-school; the school that wants to run reciprocating engines faster, and the school that wants them to go as slow as they used to go. I belong to the school that wants to run them faster. I think you will find out more if you run them faster than if you run them slower. In order to decide questions like these we often choose sides like we used to in the country when we played ball. For instance, I will take the side that wants to raise the piston speeds 500 feet a minute, and the other side can take the position of lowering them 500 feet a minute. That position ought to be easy, because we came through that place on the road up. The chairman of the meeting then asked, "Why don't we go back to 2000 feet a minute piston speed instead of going up to 3000?" After the facts were written down, we found that it would be very much better from an engineer's point of view to slow them down. But the trouble was that the people wouldn't buy the cars and that seemed to be against it. If you can convince your customer that he shouldn't run a motor fast, fine, but when you haven't anything to say about it, you have to make the motor run fast.

At the Steel Institute a few years ago, I said, "We are not interested at all in the steel formulae that you see ordinarily. They mean nothing whatever to us. We know what steel does as long as you stay within the elastic limit. That is perfectly straightforward. We are interested in what your steel is going to do after it passes the elastic limit." The members of the Institute stated, "You shouldn't use steel beyond the elastic limit." I replied, "You aren't going to build an automobile and not do that. Are you going to let a fellow drive out in the country and get stuck in a sand pit? How are you going to stop him from racing the engine and dropping the clutch in suddenly? Do you want the fellow to stay in the sand pit with a broken axle or get out? That is why we are interested in how steel performs beyond the elastic limit."

During the last five or six years, we discovered that progress occurred in reducing the weight in parts of an automobile, not in applying the ordinary formula for elastic limits, but in knowing what happens when you go beyond the elastic limit. You can reduce the sizes and increase the stresses once you decide that you have to go beyond the elastic limit. Nobody is going to keep a man from running off the end of a concrete road onto a gravel road with a good big hole in it, and you don't want anything to break, you want it to stay together. That is when you make your reputation, when the parts don't break. Thus the study of the region beyond limit became one of our problems in physics.

Going to gears, there is one very interesting thing we know. If you lay out a set of gears with the very best theory in the world, make them exactly to that dimension and put them together, there is one thing you can always guarantee, and that is they are going to be noisy. After you do that beautiful figuring, you get a paddle with a lapping compound on it, and lap them until they become quiet. What was the use of all the figuring in the beginning? There is nothing wrong with the gear theory, it is absolutely all right. The only thing is that you lay it out on a drawing board and don't take account of all factors.

Now we come to one of the most important things in all engineering, the question of elasticity. To an ordinary engineer, material has only two properties, the property of kinetic energy, which is the mass property, and potential energy, or the elastic property. The other property, a conversion factor, which we call friction enters. When looking for trouble on a machine, there are only six things that can be wrong—something can be too heavy or too light; it can spring too much or not enough; it can have too much or not enough friction. There can't be anything else wrong with it at all.

Elastic coefficients are just as basic as mass factors. For every chemical element that has an atomic number and an atomic mass, there is another equally important atomic number called atomic elasticity. It is doubtful if you will find it in any book on physics. When it comes to considering valances and chemical reactions, they are more determined by the atomic elasticity than they are by the atomic weights.

Now, what have we got to do? I don't think we have started the work of correlating industry and physics. Because after all we are all in the same boat. We are all doing exactly the same class of work. You are working here trying to push the borderline of new knowledge a little bit farther every day and that is what we are doing also. However we have to pick research up wherever we are, and we have to perform a further service for the customer, no matter who that customer may be. He may not understand your art. It might be a radio; it might be this, that, or the other thing. Therefore, we have to take these wonderful discoveries of the laboratory and put them in shape to turn over to the person who buys them; who never reads the instruction book and gets in trouble with them. The engineers used to like long instruction books. After all, the reputation of your car is never made with the first man who owns it. They always lose the instruction book before the second owner gets it and he is the fellow that determines whether the product is any good or not. Why waste the paper writing instruction books, because nobody is going to read them anyhow. The thicker the instruction book the greater the alibi the engineer has for his business. I am for very, very thin instruction books.

## III

One reason I am glad to speak to you today is that as a group you are greatly concerned with the future. Most people don't think of the future as being a definite thing, like they do of the past. The history book is definite. In fact we have been taught so much to look to the past that we are somewhat afraid to look into the future. We don't realize that what we have today has the same relation to what we are going to have tomorrow as that of yesterday has to that of today. We place a car ten years old at this end of the table and a car of today here in the middle. Then we place a car of ten years from today at that end of the table. This car of today is going to look just as funny ten years from now as that car of ten years ago looks beside the one of today.

Here is some advice I want to give to physicists: Don't be discouraged when you have a problem to solve and people tell you you can't do it. I used to talk to some of the best photographic experts and they said you couldn't make color films. Today we have them. They are a great success. So much so that last year 60 percent of all the 16 mm films were color. Fifteen percent of all the commercial pictures were all color. This year 25 or 30 percent of the commercial pictures will be in color. In about three years it is predicted there will be no black and white.

I know there are technical difficulties in doing everything. There are technical difficulties in building motor cars. We haven't a single carburetor that is right, not a single piston that is right. If we sat down and told all the difficulties that we have today some fellow would say, "I wouldn't even try it."

So far as the opportunity for young men and physicists is concerned there never was such an opportunity in the world as there is today. We are going to find out that the way we will get out of some of our social difficulties is not by legislation but by experimentation and development; by new products the people want. They have to start here in the physicists' laboratory, and you have the whole world in front of you as far as the laboratory is concerned. Just look at the things that happened in the last few years. Go back and take our automobiles 25 years ago. They laughed at them. You have the radio and telephone; they

all came along recently. Communication and transportation have gone farther than anyone believed. The first telegram was sent from Baltimore to Washington and that was a tremendous distance. Then they sent them a hundred miles and then a thousand; then the telephone came along. Then we could talk a hundred miles, then a thousand miles and across the continent. Then the radio came along, and you can talk across the world. Now, the trouble is trying to find anything worth saying that far away from home.

We are not ahead technologically. Development hasn't caused unemployment. Technological development is so far behind that we have unemployment. It's just like the fellow who sued the railroad company because the train left fifteen minutes ahead of time. The railroad company proved that the train was not fifteen minutes ahead of time, but was twenty-three hours and forty-five minutes late in pulling out. We are not ahead of the game; we are a couple of laps behind. If you are going to get men back to work we have to get on.

There are going to be physicists as long as we are going to try to push this borderline of human knowledge a little bit farther ahead. But a development is no good as long as it is in the laboratory. It is only good when everybody in the country uses it. That is the reason why we want to help you because so much depends on us. We have a fine job to do and that is the reason we have been so interested in meeting with the physicists.

---

*At the close of such a review as this, one might be permitted to indulge one's fancy in regard to the future. I confess, however, that I am not particularly good at speculation. There are some things which are certain. The first is that with the introduction into our industry of scientific knowledge and trained intelligence there cannot be any such thing in the future as a static condition. If one may so express it, the one stable thing will be continuous change. To this our minds must become accustomed and attuned. It is essential and certain, also, that systematic knowledge will be imparted to an ever-increasing degree to all who enter the industry, whilst research will gradually become an essential part of every factory organization. In our own Society, we hope to be sustained, in the future as in the past, by men who hold high rank in leadership of the new knowledge. But whatever other qualifications they possess, we know that it is "the spirit which quickeneth;" and that when, in a cooperative movement such as ours, personal ambition is generously tempered by a genuine desire to be of service, the future of the Society's work and the objects for which it was founded will certainly be assured.*

W. E. S. Turner

*A Professor Looks Out on the Glass Industry*

# *Resumés of Recent Research*

**Recent advances in experimental and theoretical physics are described in these columns in nontechnical language. It is intended that sufficient background be given to introduce the reader to the subject.**

**Systematic Study of the Induced Radioactivity of Rare Earth Elements**

That very interesting group of elements in the periodic classification known as the rare earth group (elements 57–71 inclusive) has been the object of a systematic study in induced radioactivity produced by bombardment with fast and slow neutrons.*

The study was inhibited, as is most work on these elements, because of the usual difficulties associated with rare earth research. Some of these elements, e.g. holmium, can be had in only small quantities. This complicates obtaining accurate measurements of half-life periods for those elements which show a very weak activity. Because of the extreme similarities in the chemical and physical properties of these elements, separation of one from the others is long and tedious. This makes it impossible to carry out chemical separations to prove certain radioactive transformations except in one or two instances. In the case of cerium a chemical separation can be effected rapidly enough to detect radioactive products in separate fractions. Many of the active periods observed are short or weak in intensity, which renders radioactive studies more difficult.

Forty-four stable isotopes of the rare earth elements have been reported. Twenty-seven radioactive isotopes have been detected, of which twenty-five were measured by the authors. In all, nineteen instances of electron emission and six of positron emission have been recorded for these elements. It is of very definite interest that in this study seventeen nuclear reactions in which neutrons of high energy are "boiled out" were noted. That is, seventeen $n-2n$ nuclear reactions are manifested by the rare earth elements.

Dysprosium may be considered an excellent "neutron detector," since this element forms the strongest slow neutron period yet observed for any element. Europium, likewise, is easily activated, whereas lanthanum and cerium are but weakly radioactive even after being strongly irradiated.

Erbium proved to be of special interest since the strong 1.1 min. positron period distinguishes erbium (165) as being the heaviest element in the periodic classification yet known to emit positrons.

Of particular note is the high neutron equivalent of the cyclotron at Michigan. This is very evident in comparisons of activity produced in neodymium by neutrons from a radon-beryllium source, and from beryllium bombarded by deuterons produced by the cyclotron. For example, Marsh and Sugden did not observe any induced activity with a 400 millicurie, radon-beryllium source, whereas, as reported in this last study, three periods were easily evident after four hours of slow neutron bombardment with neutrons from beryllium activated by deuterons.

Evidence for new isomeric isotopes was noted in two instances. Gadolinium (at. wt. = 159) can be activated to give a 3.5 min. and a 17 hr. period. Likewise, ytterbium (at. wt. = 175) is an isomeric isotope, giving two periods, one of 2.1 hr. and another of 14 hr. after being subjected to neutron bombardment.

The necessity of postulating the existence of four new stable isotopes, gadolinium (152),

* A paper on this subject by Professors M. L. Pool and L. L. Quill of Ohio State University appeared in a recent issue of the Physical Review (**53**, 437 (1938)).

dysprosium (160), erbium (164), and ytterbium (170), resulted from studies of the radioactivity data. Dempster, with his mass spectrograph, has since been successful in detecting these isotopes as well as four other new isotopes of the rare earths.

The field of induced radioactivity, therefore, is stimulating further an interest in isotopic studies with the mass spectrograph.

---

**New X-Ray Diffraction Techniques**

The technique of taking x-ray diffraction photographs of metals or other materials has recently been improved by several workers in this field. Guinier,[1] in France, has used the scheme shown in Fig. 1 (solid lines). X-rays fall upon a

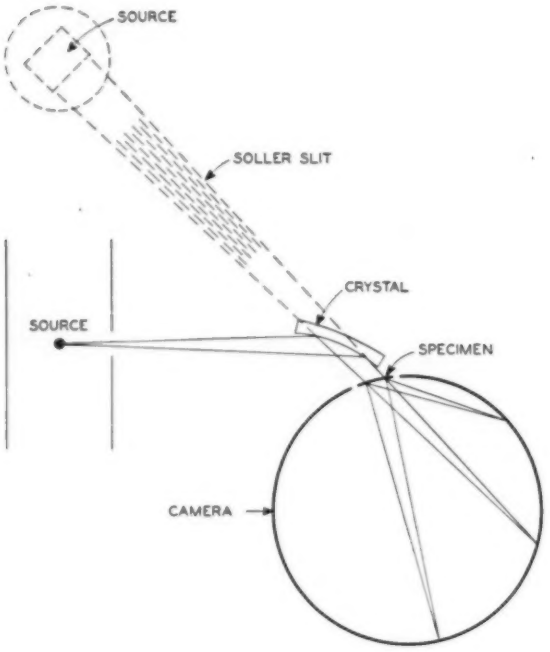


FIG. 1.

crystal of quartz which is cut and bent[2] so as to bring the beam to a focus that is made to lie on the circumference of a camera which holds the specimen and the photographic film. The beam is thus monochromatized and concentrated, and the geometry is such that all diffracted beams from the specimen are focused on the film (Seemann-Bohlin focusing).

Frevel[3] has used a Soller slit, containing 24 lead diaphragms, to focus an x-ray beam from an extended source. As shown in Fig. 1 (dashed lines, specimen and camera, crystal removed), the rays fall on the specimen and are diffracted and focused as in the arrangement of Guinier.

Bozorth and Haworth[4] have used the arrangement of Fig. 2, which was suggested by Guinier also. Their original contribution is the use of a crystal of rocksalt instead of one of quartz. While the cutting and elastic bending of a thin quartz plate is a laborious job, rocksalt may easily be bent plastically under water to almost any desired form. Bending to one radius of curvature and grinding the surface to another radius is accomplished in a few minutes time, and the crystal maintains its form without the application of stresses. Also, the intensity of the beam focused by rocksalt is greater than that focused by quartz.

By using the several arrangements described, powder photographs with high dispersion have been obtained in exposure times of one to five minutes. On account of the efficient monochromatization of the reflection from the curved crystals, the photographs taken with their aid are unusually free from any diffuse background, so that longer exposures may be taken to bring out faint lines.

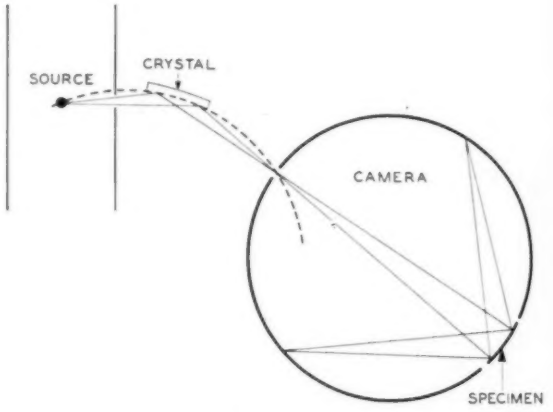


FIG. 2.

[1] A. Guinier, Comptes rendus **204**, 1115–6 (1937).
[2] J. W. DuMond and H. A. Kirkpatrick, Rev. Sci. Inst. **1**, 88 (1930). T. Johansson, Zeits. f. Physik **82**, 507 (1933).
[3] L. K. Frevel, Rev. Sci. Inst. **8**, 475–7 (1937).
[4] R. M. Bozorth and F. E. Haworth, Phys. Rev. **53**, 538–44 (1938). [The paper of Guinier was unknown to these authors, and the abstractor (R. M. B.) takes this occasion to acknowledge Guinier's work.]

# Human Beings and the Motor Car*

BY CARL BREER

*Director of Engineering Research, Chrysler Corporation, Detroit, Michigan*

HUMAN beings and motor cars are among the most competitive things we have in existence today. Consider the cave man, for instance. We see him moving about from place to place gathering food from Nature's supply for the family. He is alert, he is fast on his feet, and he uses his arms advantageously in many different ways. He has developed all of his natural traits and protective senses to a highly efficient degree. When we look at him structurally, we see one of Nature's most wonderful pieces of mechanism, consisting of levers of bone which are directed in operation by senses, and operated by sheer muscular strength. His legs are used mainly to move from place to place. His hands, more refined, with multiple levers in the forms of fingers, are useful in doing mechanical work in his immediate environment.

We can picture the thrill this primitive man experienced when he discovered that by means of the club he could pry things loose and do many things that without this lever he could not accomplish. Think also of the thrill of discovering that if he built this lever in a continuous form about a pivot he had a wheel. What a revelation! Today we mark this accomplishment, one of the important points in the advancement of civilization.

And yet, at this very moment, engineers wonder in amazement about the present endeavor to adopt man-made laws to limit and restrict, or heavily tax, labor-saving devices. What cruel selfishness of mankind. The cave man would have used better judgment.

Taking full advantage of our worldly endowment of elements and using God's fundamentals to our best knowledge, we have, in the rapid progress of civilization, advanced to the present stage of the four-wheel iron horse, the motor car of today. When we think in terms of what caused us to develop the motor car, we return to our human desire of wanting to move about in a more efficient manner. The horses that we had were not dependable and did not fully satisfy our wants; so man made a machine to substitute for the horse, and a new thrill greeted us—another step in the advancement of civilization was taken.

Let us now outline the development of the present automobile; it was first necessary to make it run; second, it was essential to make the mechanism endure; third, to make it quiet; and, last and all-important, refine it and make it a comfortable companion. Before we reached this latter stage we suddenly awakened to the fact that we had been selling automobiles to human beings, and in effect, had been saying:

"Here is our car. You will like it. Ride in it and, if it jolts you around, you will soon adapt yourself to its inequalities."

We had considered the motor car from an entirely wrong point of view. Today, we know that human beings are fixed entities and cannot be changed.

So, we began to think of the evolution of the cave man and life. We thought in terms of this early man walking from place to place for many generations and we discovered that during this period Nature built up a most efficient human machine in proper balance in every respect. This beautiful mechanism is a structure of mechanics that we all admire and respect. Thermody-

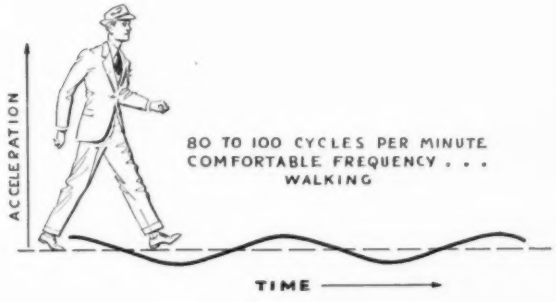


FIG. 1.

* Paper presented at the Symposium on Physics in the Automotive Industry at Ann Arbor, Michigan, March 14 and 15, 1938.

namically, mechanically, dynamically, and in every way, it is perfect, and we will continue to marvel about it.

So, with due consideration, we study Nature's own action. We realize that over a period of time man has established a comfortable stride without fatigue. Specifically, we find this to be around $2\frac{1}{2}$ to 3 miles per hour. In terms of engineering, we think of it as 80 to 100 cycles or steps per minute. Let us call this the human comfort range. We notice further that the heart, the kidneys, the liver, etc., are all suspended in the body cavity by means of muscular chords. Nature has established a natural rhythm at which the whole human mechanism can be most comfortable.

We then tried finding what human reaction would result if we built a motor car with a riding frequency that coincided with a comfortable walking rhythm. Cars previous to this time were stiffly sprung. American cars ran with a frequency range of 120 to 150 cycles. European cars had even higher rates, from 150 to 200 cycles per minute. Of course, mechanically, one of the problems was how to do this commercially. In our endeavor to accomplish this result, we rearranged the car's weight distribution and by so doing increased the moment of inertia with respect to the car body longitudinally. In order to get the desired result, we also established centers of percussion points close to the axle centers.

A mechanic likes a particular hammer because the handle has no sting. Basically, we say it was properly designed because it was held at center of oscillation when striking on the center or percussion. Another example we have all experienced is the sting of a baseball bat when hit off center of percussion. The most interesting part about the simple fundamental of relation between centers of oscillation and centers of percussion is that they are interchangeable. With the application of this fundamental principle we immediately slowed down or practically eliminated what we call secondary oscillation with respect to the front and rear axles.

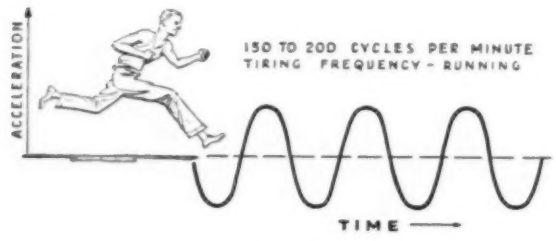


FIG. 2.

Digressing for a moment, it is very logical to think in terms of what Nature has done for animals in applying its own fundamentals of the relative foot action with respect to centers of percussion. You can picture a race horse with its neck straight out and its jockey in a position to give his horse maximum efficiency in the use of his limbs. The fleetest horse is the one with better balance between mechanical mass and force. If forces are not acting with proper respect to centers of percussion, then energy is wasted, creating inefficiency. In racing, it was discovered that a jockey could speed up his horse and win races by riding forward and hugging the animal. All he was doing was helping the mass action to give the horse greater efficiency with respect to centers of percussion.

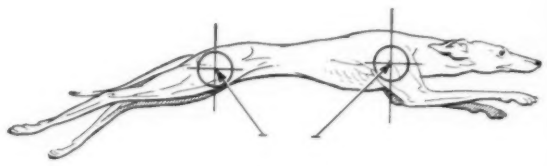

FIG. 3. Location of centers of percussion in a running grayhound.

A little while ago some of my associates checked the mechanics of a four-legged animal. They arbitrarily decided to experiment with a Russian wolfhound, because it is one of the fastest dogs we have. They were fortunate in obtaining a blue ribbon winner, an excellent specimen. Upon studying the data, they found that the dog's legs joined the body exactly at its centers of percussion.

In the language of the physicist, I think it would be extremely interesting to study the natural history of animals with respect to mechanics and dynamics as Nature has applied them. Also, it would be interesting to establish better limits with respect to human comfort range as to vision, hearing, smell, taste, and other senses of human life.

The experiments on weight distribution proved so outstanding that it started a whole new

evolution in motor car engineering. They established a new practice—that of designing a motor car with thought of a fixed entity of the human being. The moment we established this principle, we took a new viewpoint with respect to cars. I think you all remember a few years back, when you went on a trip, you would argue as to who was going to ride in the front seat, for the simple reason that the rear seat ride was terrible. You feared being pitched into the roof when passing over a bump. Today, because of consideration of this new principle of center of percussion and proper spring periodicity, with proper shock absorption, you have forgotten about your desire to ride in the front, and often you find it preferable to ride in the rear seat.

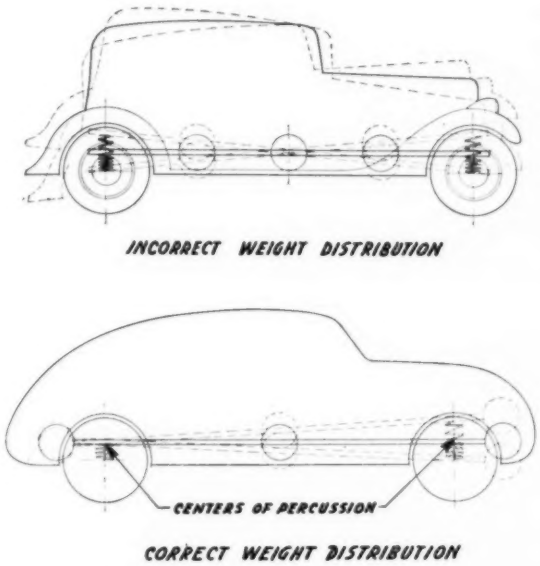


FIG. 4. Diagram of spring action.

Following through with the application of Nature's fundamentals to motor car design, we picture a car functionally. The external shape of an automobile as to future trend is certainly functional with respect to the elements to which it is exposed. Externally, the design will move in the direction of increased aerodynamic efficiency. The car suspension must function to furnish the maximum comfort and safety to passengers. The external shape is also functional with respect to passing motor cars. If we had overhanging sides rather than extended fenders, etc., in the hands of the average driver, there would be more hazard and more damage.

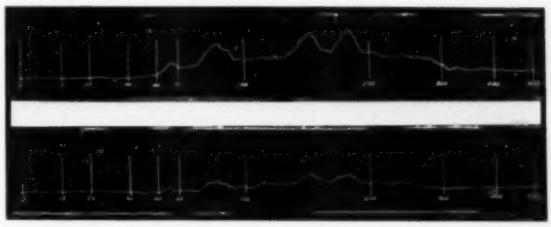

FIG. 5. Sound frequency spectra. Typical curves showing the intensity of road noise at frequencies from 10 cycles per second to 500 cycles per second. Upper curve before sound treatment, lower curve after treatment.

As to the interior of the car, the factors of first importance are the driver's and occupants' comfort and safety. We continually keep the occupant in mind, and try to subdue and overcome the things that annoy him most. Noise is a factor and a study all its own. Noise reaction is a thing that must be evaluated as to its effects on the human mechanism. Steering, clutching, braking must be within the comfort range of the average driver's muscular activities. Seating comfort is a matter of buoyancy. We have steering wheels that will resist collapse by bending instead of breaking in case of accident. We eliminate projecting gadgets as far as possible. There are many other factors that must be balanced with regard to safety, comfort, and fatigue.

With further regard to the human equation, we try to use foolproof devices wherever possible. With carburetors, we used to furnish adjustable jets. Today, they are fixed so as not to be tampered with. Starting is now done by automatic choke, taking away the driver's opportunity to do the wrong thing. In a visit to the plants, you will see many of the unit parts of the cars that have been designed purposely to prevent human error in assembly.

Truly, progress in industry, in the eyes of science, is discouragingly slow. This is caused by mass action of a multitude of minds to satisfy. In large organizations, you have minds of vision, minds that are obstinate, minds that are enthusiastic, and minds that are conservative. Put them all together, and the result, after eliminating all

the things that are not to be done, is that your product comes out as the best composite result. This is as it should be.

We endeavor to build the best piece of mechanism we know how. If it were possible to deliver a car tomorrow that would run hundreds of thousands of miles without a repair or adjustment, and would require only the addition of oil, water, and gas, we certainly would do it. We would have no fear of the saturation bugaboo for the simple reason that we expect to make sufficient future progress to obsolete the present motor car. Drive our present-day cars and then one of the models three years old, and you will immediately realize what we mean.

Every now and then we hear talk of revolutionary change across the horizon of the motor car world. It might be a rear engine motor car, or a Diesel driven vehicle. Such sudden changes have not happened in the past and common sense with a little reasoning will tell you why.

As to rear engine cars, by simple reasoning you can draw your own conclusions. Diesel propulsion of a motor car is a matter of the best commercial compromise. Here we eventually face basic economy of fuel costs. With modern processes, added refining cost per gallon of gasoline above that of Diesel fuel is between one-half and one cent. Add gas station distribution and taxation, and the cost of the fuels would be reasonably the same. Hence, the basic problem becomes one of commercial balance between added first cost and increased economies of added miles per gallon, assuming flexibility, performance, and other things as being equal. The automobile is one of the most vulnerable things on the market. Competition is keen. There are no price fixing agreements. The underlying code is very simple. Can we out-engineer or out-manufacture and out-sell our competitors? The respect we have for each other is all good, clean-cut sportsmanship. Every year, instead of a New Deal, we produce a new deck of cars, and the game is played openly on the table of public acceptance.

Speaking of research, we once thought it would be ideal to have a yearly appropriation, say a million dollars, to follow research endeavor. How would we set up a research program? Perhaps the first thought would be, set up an organization and measure it by a detailed division of the mythical million. Next, go about looking for ideas. On this basis, when the year was ended, and the money gone, the ever methodical and hard-boiled accountants would step in and say: "You did a swell job of spending, now what have you got to show?" Fortunately, good research work isn't done this way, yet many a fortune endowment has been wasted on this hypothesis.

On the other hand, if an idea of merit is placed before us by one of our own men or an outside organization, our own judgment is the determining factor for spending money on preliminary analysis. We have, in unusual cases, taken money out of our own pockets to satisfy our curiosity. If the project has merit, development work proceeds and results are measured accordingly, with every intention of forcing the project to improve our motor cars. Major research is seldom accomplished overnight, but often requires periods of six or seven years.

In research one problem is to find the type of men with proper incentive, initiative and balance of enthusiasm, and good judgment. We prefer the fellow who strives to find how it is possible to accomplish the desired result, rather than the negative individual whose reaction is to establish the many reasons why it cannot be done. In every organization, we have many more pessimists than optimists. As a result, real research men are very scarce.

FIG. 6. Apparatus used in choosing colors in an automotive research laboratory.

Fundamentals long established by physicists are our guiding rules in automotive engineering. As to future research problems, the physicist primarily is interested in fundamental phenomena. The automotive industry, we feel, parallels most industries in being interested in commercial research leading up to the borderline of fundamental research. This is natural because fundamental research so often starts with an objective far removed from any particular industry.

There is no doubt that there are many things in the physicist's mind which might be applicable to our industry if the physicist could see at closer range our scene of action. Hence, we gladly encourage the physicist to become better acquainted with the commercial and economic considerations of industry. And likewise, it is necessary for industry to have a better appreciation of what is in the mind of the physicist. It is only by virtue of honest intent and cooperation that any modern industry can survive.

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*Forward-looking labor organizations have shown appreciation of their debt to science. For example the 1919 Convention of the American Federation of Labor adopted Resolution 94, from which the following is quoted:*

*"WHEREAS, The increased productivity of industry resulting from scientific research is a most potent factor in the ever-increasing struggle of the workers to raise their standards of living, and the importance of this factor must steadily increase since there is a limit beyond which the average standard of living of the whole population cannot progress by the usual methods of readjustment, which limit can only be raised by research and the utilization of the results of research in industry;*

*"RESOLVED, By the American Federation of Labor in convention assembled, that a broad program of scientific and technical research is of major importance to the national welfare and should be fostered in every way by the Federal Government, and that the activities of the Government itself in such research should be adequately and generously supported in order that the work may be greatly strengthened and extended; and the Secretary of the Federation is instructed to transmit copies of this resolution to the President of the United States, to the President pro tempore of the Senate, and to the Speaker of the House of Representatives."*

K. T. COMPTON

# Physics of Rubber as Related to the Automobile*

BY W. F. BUSSE

*Physical Research Laboratory, The B. F. Goodrich Company, Akron, Ohio*

## *I. Introduction*

THE automobile is largely responsible for the development of the rubber industry. Rubber, in turn, has made such important contributions to the safety, comfort, efficiency, and quietness of the modern automobile that an automobile without rubber is almost inconceivable. Even the thought of traveling over highways at present-day speeds with the iron tires of the horse-and-buggy days makes us shudder. Rubber tires not only cushion road shocks and withstand abrasion, but they also make steering possible. In many other less obvious places rubber products play an equally essential role. I need only mention fan belts, which may transmit loads up to 10 or 15 hp at linear speeds over a mile a minute, motor mountings which isolate the vibration of the motor from the rest of the car, insulation strips between metal parts of the body to prevent squeaking, hydraulic brake hose, running-board covers, windshield wipers and sponge rubber or rubberized hair (Nukraft) cushions in upholstery, to illustrate the host of other minor but essential uses. It is reported that rubber is used in some 180 places in the modern car.

The many different uses for rubber depend in nearly every case on its unusual *physical* properties, and on the combinations of physical properties that can be obtained by modifying the rubber and by combining it with fabrics and metals in various mechanical structures. This has been recognized to the extent that physical tests are used almost universally to judge the quality of rubber goods. However, it is only recently that any intensive effort has been made by physicists to relate the gross physical properties of rubber to the physical and mechanical properties of the rubber molecules. This work already has yielded a surprising amount of very practical, as well as fundamental scientific information. In this paper I will try to show by a few examples how this new information, as well as the older fundamental laws of classical physics, are being applied to improve the old and develop the new rubber products.

The problems of the physicist in the rubber industry may range from those bordering on pure chemistry to those bordering on pure mechanical engineering or mathematics; and in certain problems concerned with viscosity and with infrared spectra, there is even the possibility that there may be a use for the quantum theory. In general, the problems fall in three groups:

1. Processing problems. These include: studies of the flow properties of rubber pigment systems having both elasticity and plasticity; heat conduction problems; statistical studies of variability of raw materials, processes and products.

2. Fundamental studies of the physical and mechanical structures of the rubber molecules.

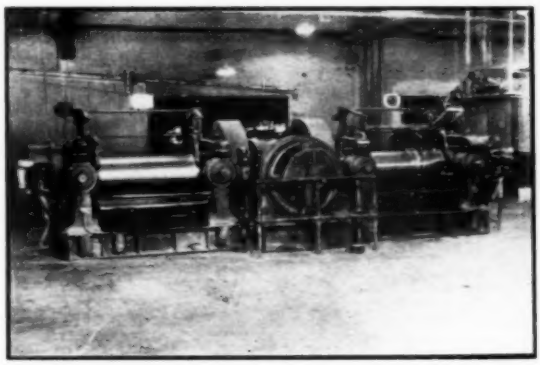


FIG. 1. Double 20″×60″ mill unit mounted on rubber springs.

* Paper presented at the Symposium on Physics in the Automotive Industry at Ann Arbor, Michigan, March 14 and 15, 1938.

3. Development of new and more efficient rubber products. This often involves finding better methods of combining the high elasticity of rubber with the rigidity and strength of other structural materials such as fabrics and metals.

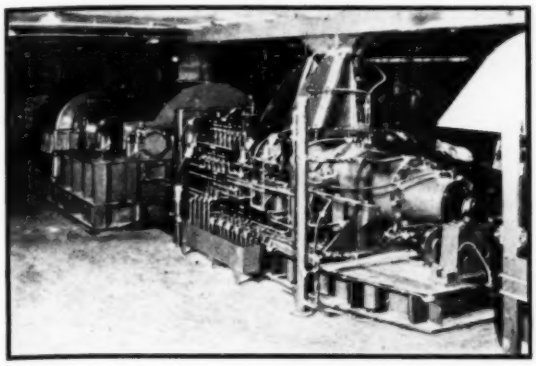


FIG. 2. 20" two-stage plasticator unit mounted on rubber springs.

In this connection it is curious that the one property, high elasticity, which probably is more responsible than any other for the present wide range of uses of rubber, both makes possible and hinders certain new uses.

The average person who is used to dealing with inherently rigid materials such as metals, wood and stone, instinctively associates strength with rigidity. We are inclined to feel that a material which can be stretched 600 percent to 800 percent before failure must be inherently weak and cannot be trusted to carry loads even if the stress required to produce the failure may be nearly 5000 lb./in.$^2$, calculated on the original cross section, or as much as 30,000 to 40,000 lb./in.$^2$, calculated on the cross section at break.

The skepticism concerning the reliability of rubber may have been partly justified in the past, not because of its high elasticity, but because of other properties such as low tear resistance, rapid deterioration or excessive creep. Today, however, compounds have been improved, and their limitations as well as their advantages are known more accurately. In addition the mechanical design of rubber mountings has been improved, so that now rubber in shear can be used to support the entire static and dynamic load of large industrial machines weighing up to eighty tons, or to replace steel springs on street cars. It is now even possible to use rubber torsion springs to replace the springs of automobiles.

### *II. Manufacturing Processes*

Crude rubber as it comes from the plantation is a tough elastic material that becomes hard on cooling, and soft and tacky with moderate heating. Charles Goodyear discovered about one hundred years ago that if the rubber were mixed with sulfur and heated (vulcanized) it retained its elasticity over a wide temperature range.

Some thirty years ago, Mr. George Oenslager, a chemist who is still with the B. F. Goodrich Company, discovered that the reaction of rubber with sulfur could be hastened and the quality of the rubber improved by the use of organic materials which are now called "accelerators." It was found that the abrasion resistance of rubber can be greatly increased by incorporating with it large quantities of carbon black. Then age resisters were developed which greatly reduce the deterioration of rubber. During recent years chemists have developed hundreds of accelerators and age resisters and compounders have made many thousands of rubber compounds with a wide range of properties.

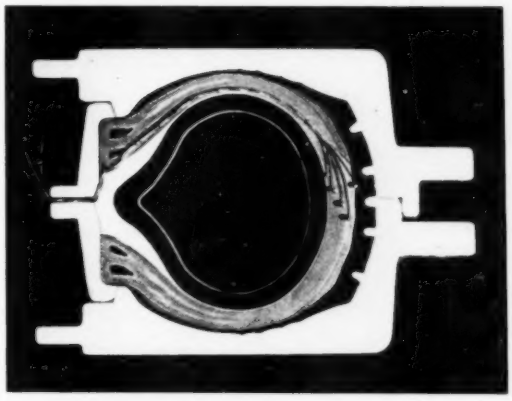


FIG. 3. Cross section of tire and water bag in mold showing location of thermocouples.

The first step in making most of these compounds involves masticating the rubber between large mill rolls such as are shown in Fig. 1, or in a plasticator, such as that shown in Fig. 2, which is built somewhat like an enormous sausage

grinder. This operation changes the rubber from a tough, elastic substance to one which is soft and plastic. It is then possible to mix thoroughly the necessary solid pigments such as gas black, sulfur, accelerators, and age resisters with the rubber while it is being worked on a mill, or in an internal mixer. It was thought for a long time that the breakdown involved a mechanical disintegration of the rubber, but it is now known to be due to an oxidation process.

Proper control of the rubber manufacturing processes requires a quantitative measure of this complex property, plasticity, which involves both elastic deformation and viscous flow, both of these factors, in general, being functions of time and of rate of flow. Physicists have made important contributions to the methods of measuring these factors, but the measurements still are necessarily empirical, and much remains to be done on the general problem of the flow of visco-elastic systems.[1]

FIG. 4. Tire mold in heater showing thermocouples coming out to switch and potentiometer.

[1] It may be noted that for a long time theoretical physicists have ignored almost completely the problem of the cause of, or the mechanism producing the viscosity of simple homogeneous liquids. No theory of the viscosity of liquids gave even the proper sign for the temperature coefficient of viscosity, and, except for the almost isolated attempt by Maxwell to explain the elastic after-effects of solids, the effects of elasticity on flow were completely ignored. In the past few years Dunn (1925), Andrade (1934), and others have developed theories that give both the correct sign and magnitude of the changes in the viscosity of simple liquids with temperature and these ideas have been used in studying the state of rubber in solution. Eyring (J. Chem. Phys. **4**, 283 (1936)) has attempted to work out a more general theory of viscosity on a quantum-mechanical basis, which may also explain some of the elastic properties. Rubber solutions should be ideal systems for testing this theory, since they can be varied in concentration from 0 to 100 percent, with over $10^8$-fold changes in viscosity and probably equally great changes in elasticity. (See Ewell, J. App. Phys. **9**, 251 (1938)).

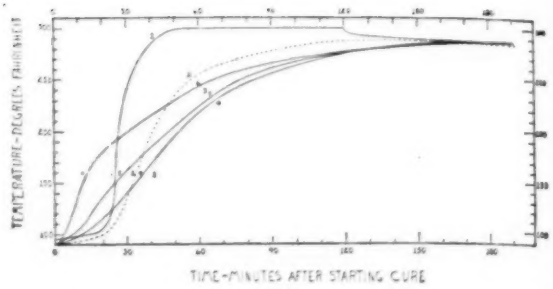



FIG. 5. Temperatures within a tire during "hot water" cure. Couple No. 1, at surface of tread. Couple No. 2, at base of tread. Couple No. 3, at top of 12th ply. Couple No. 4, at top of 8th ply. Couple No. 5, at top of 4th ply. Couple No. 6, at surface of inner ply.

The soft, plastic mixtures of rubber and compounding ingredients are of only limited use until they are cured or vulcanized by heating under controlled conditions. The physical properties of the final compounds, as well as their resistance to aging, depend to a large extent on the temperature to which the rubber is heated, and the time it is held at that temperature. Because rubber is comparatively a poor conductor of heat, the problem of getting a uniform heating of thick objects is not an easy one.

In some cases, as with long rods or flat plates, the rate of heating may be calculated from theory, provided the diffusivities of the compounds are known. Hence it is important to have accurate measurements of the specific heat and conductivity of rubber and pigments such as sulfur, zinc oxide and carbon black.

Most rubber objects are so complicated in shape and construction that it is impossible to calculate their rates of heating from theory, and actual temperatures at different places in the sample must be measured at various times during the cure. This usually is done by building thermocouples into the object such as a tire and measuring temperatures with a potentiometer. Fig. 3 is a cross section of a tire and water bag in a mold, showing the distribution of the hot junctions through the tire. The first junction is at the surface of the tread, the second is immediately under the tread, and the sixth is at the inner surface of the tire next to the water

bag, with the others at the 4th, 8th and 12th plies.

The mold containing the tire and water bag is put in a heater which can be filled with steam under pressure, the thermocouple leads being brought out of the heater through a special packing gland. Fig. 4 shows the mold in the heater and the thermocouple connection to the potentiometer. Fig. 5 shows typical curing curves obtained. Hot water under pressure is first put in the inner water bag for 20 minutes.

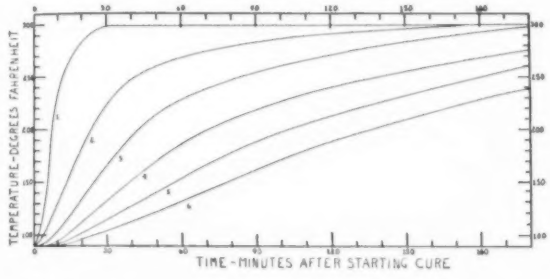



FIG. 6. Temperatures within a tire during "air bag" cure. The location of the couples is the same in Fig. 5.

This heating under pressure makes the tire conform to the grooved inner surface of the mold and allows the heat to pass through the water bag to the inside of the tire. Steam is then applied outside the mold causing a rapid temperature rise at the surface of the tire. The temperature rises more or less uniformly in the other plies, and at the end of 120 minutes the external steam temperature is reduced, partially cooling the mold. The tire is removed at 180 minutes. It will be noted that all parts of the tire receive nearly equivalent cures.

If, instead of using this "hot-water cure" one tried to heat the tire entirely from the outside and used only compressed air in the inner bag to force the tire into the grooves of the mold, heating curves similar to those shown in Fig. 6 might be obtained. Here the inner plies of the tire do not come within 60° of curing temperature in three hours. Under these conditions the various plies would have to contain compounds of different rates of cure, those on the inside curing at the lowest temperature, and the outer plies curing at progresively higher temperatures. This would, of course, involve considerable difficulty from a production standpoint, making the "air bag" cure less satisfactory than the hot water cure.

### *III. Structure of Rubber*

Not very much is known about the chemical reactions that occur during vulcanization except that they usually involve some combination of rubber with sulfur. The effects on the physical properties are very striking, however, and they give some clews to the change which takes place in the molecular structure.

The uncured rubber compound has almost no strength, yet after cure it may have an ultimate tensile strength of 4000 to 5000 lb./sq. in. and an elongation of 600 to 800 percent or more, as shown in Fig. 7. Since the rubber decreases in area almost in direct proportion to the elongation, the strength of the rubber may be around 40,000 lb./sq. in. if calculated on the cross section at break.

Another very striking change produced by vulcanization is in the behavior towards solvents. Uncured rubber dissolved in benzol to form a viscous solution, while well-cured rubber may swell to 1000 percent of its initial volume, without losing its shape.

Staudinger and others have shown that the viscosity of a material in dilute solutions may give at least some indication of the molecular

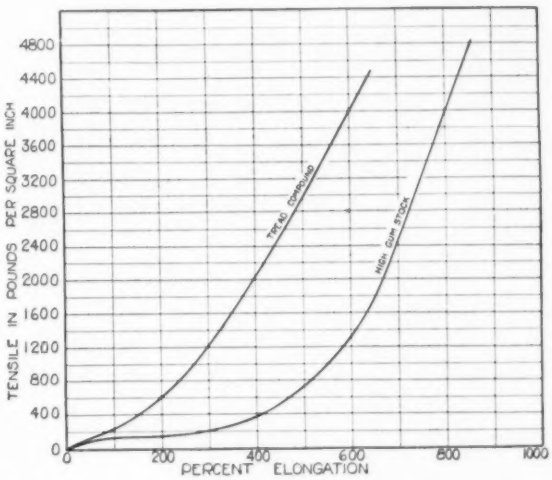


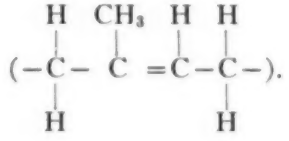

FIG. 7. Stress-strain curves of tread compound and of high gum compound.

weight. As a result of viscosity and osmotic pressure tests, it has been shown that crude, unvulcanized rubber is composed of long, thread-

like molecules having molecular weights of the order of 10,000 to 100,000 or more.

When molecules become this large chemical methods of studying them are very unsatisfactory. For instance, to detect a chemical reaction involving the combination of one molecule of rubber of molecular weight 100,000 with one atom of oxygen or with one atom of sulfur would require tests sensitive to 0.016 percent combined oxygen or 0.032 percent combined sulfur. Even these tests would not tell much about where or how the oxygen or sulfur had combined.

As the molecular weight increases certain physical properties which depend on the $\frac{1}{2}kT$ erg of translational kinetic energy per mole, such as vapor pressure or osmotic pressure, also become relatively unimportant. However, since the mechanical properties of these large molecules such as elasticity, or swelling in solvents, become very sensitive to changes in structure, mechanical tests are more important in studying the molecular structure than chemical measurements. The chemical data available today suggest that the uncured rubber molecules are made up of a great many rearranged isoprene units



Physical tests suggest that they form long chain or thread-like molecules. Vulcanization then probably involves forming cross bonds between the different chains.

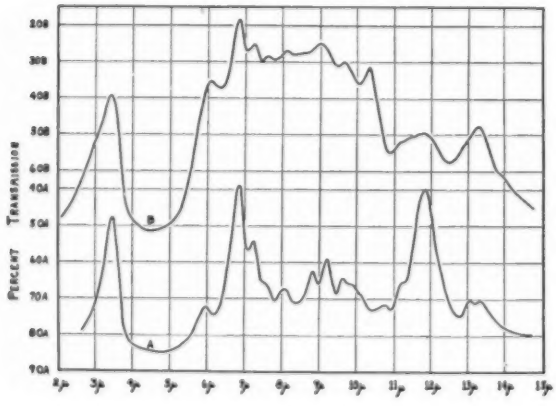
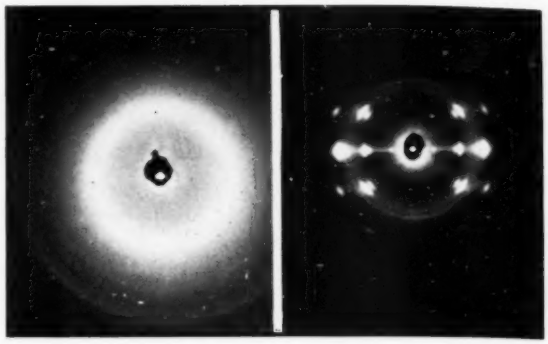



FIG. 8. Infra-red transmission spectra of crude rubber (a) and ebonite (b).



(a) (b)

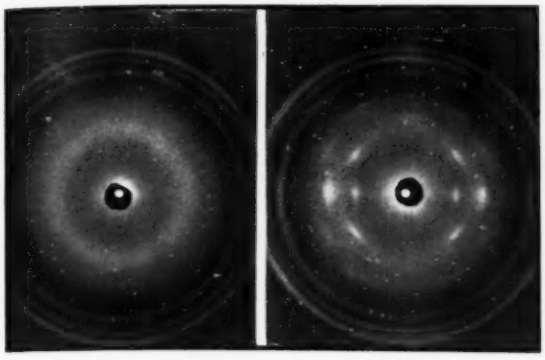
FIG. 9. X-ray diffraction patterns of a high gum compound. (a) Unstretched; (b) stretched 600 percent.

Certain fundamental conclusions about the structure of rubber can be drawn from purely geometric considerations. Rubber can be stretched nearly 1000 percent, without an appreciable change in density, and it will regain substantially its original shape when released. In this process, on the average, the $x$ components of the distances between atoms in the rubber are increased tenfold, while the $y$ and $z$ components are each reduced by the factor $(10)^{\frac{1}{2}}$. It is known from band spectra and crystallographic data that atoms themselves do not have this deformability, and that any tenfold, or even two- or threefold increase in the distance between atoms held together by either polar or homopolar bonds would inevitably result in rupture.

It follows that rubber must contain some mechanical structure which magnifies the slight extensibility and deformability of single atoms. Such a structure could be formed by holding atoms together by strong forces in long chain molecules which were surrounded by weak or uniform secondary valence forces. This would allow the molecules of rubber to slip past each other easily when they were deformed.

It may be noted that the requirement of long chain molecules is met by many materials such as cellulose, certain resins and polystyrol; but these materials are not elastic under ordinary conditions because the secondary valence forces around these molecules are too high. When these

secondary valence forces are isolated or insulated by adding small molecules (plasticizers) which



(a) (b)

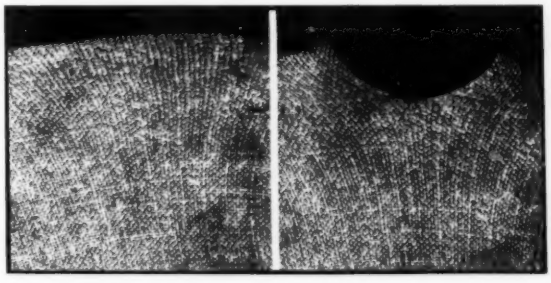
FIG. 10. X-ray diffraction patterns of a tread compound. (a) Unstretched; (b) stretched.

are adsorbed on the fiber molecules the materials become elastic. Polyvinyl chloride, for example, is a hard, horny material, but if 60 parts of polyvinyl chloride are heated and pressed with 40 parts of tricresylphosphate a very elastic but tough material (Koroseal) is formed.

The remarkable thing about the elasticity of rubber is not the great extension but rather the nearly complete recovery when the external stress is removed. This requires that the long, thread-like molecules be tied together in a few places along their length to form some sort of a three-dimensional network. The stretching force then lines up the atoms in a more or less parallel array, decreasing the disorder, or entropy, of the system. On removal of the external stress, the kinetic vibration of the atoms within the chains tends to restore the state of disorder or maximum entropy, in which the chains have irregular, zigzag shapes. Because the chains are interlocked into a network, the contraction of the chains into a zigzag shape brings the whole sample back to substantially its original shape.

Because this type of deduction gives only a broad, general picture, the detailed information about the structure of the mechanical rubber molecules must be obtained by means of other physical tools. One that is of particular interest is infra-red spectroscopy. Fig. 8 shows the infra-red transmission spectra of crude rubber and of ebonite $(C_5H_8S)_n$, obtained by Dr. Sears of our laboratory while at Ohio State University. The larger peaks can be identified as due to vibrations of atoms or small groups within the molecules, such as $\nu(C-H)$ at 3.4 mu and $\delta(CH_2, CH_3)$ at 6.86 mu. One might attribute the 11.8 mu band in crude rubber to C−C vibrations and the 6.0 mu band to C=C vibrations, but the latter band also appears in ebonite which is not supposed to have any double bonds and the 11.8 mu band is almost suppressed in ebonite, although it should have just as many C−C bonds as does crude rubber. The spectrum of soft vulcanized rubber out to 15 mu does not differ appreciably from that of crude rubber. The infra-red data agree quite well with the data from the Raman spectra of rubber.

X-rays have thrown considerable light on the structure of rubber. Fig. 9a shows the amorphous pattern of an unstretched sample of cured rubber containing only a small amount of pigment. When this sample is stretched 600 percent, the beautiful fiber crystalline pattern shown in Fig. 9b is obtained. Similar effects can be obtained with crude rubber, but the crystallization is not quite so perfect. The pattern of a heavily loaded tread compound is shown in Fig. 10. The inner broad diffuse band is due to amorphous rubber



(a) (b)

FIG. 11. Strain distribution in rubber during tear. (a) is before a tear starts. (b) is after a cut or tear is put into the edge of the sample.

while the outer narrow bands are due to pigments, especially zinc oxide. The orientation of the rubber crystallites produced by stretching this compound to 600 percent is less perfect than in the high gum compound. It will be noted that there is a tendency for the zinc oxide particles to orient in the direction of stretch. These pictures

were taken by Professor Clark at the University of Illinois.

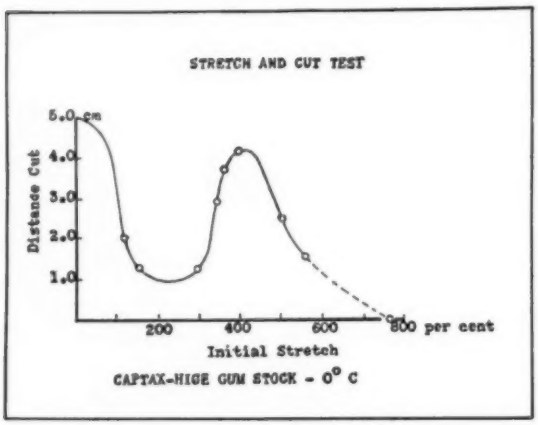



FIG. 12. Effect of initial stretch on length of cut necessary to start a tear.

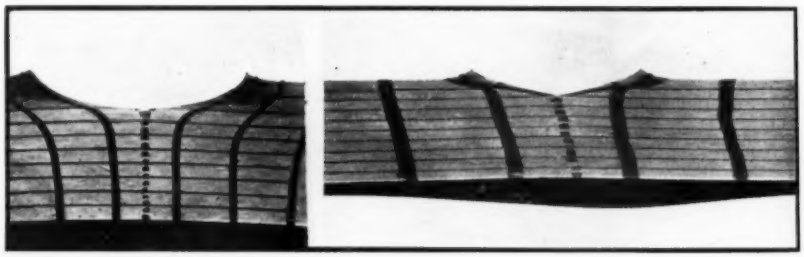
The crystallization of rubber on stretching accounts for a number of peculiarities in its gross physical properties. A sample of crude rubber which is stretched and released normally snaps back to its original dimensions. However, if the sample is cooled a few degrees below room temperature while it is held stretched, it retains most of its elongation when the deforming force is relieved—that is, it becomes "racked." On warming it returns to its initial size. The racked rubber not only shows fiber crystallites with x-rays, but it also has a mechanically fibrous structure, as can be shown by tearing it.

During stretching, rubber gives off considerable heat, and on contracting, it cools. This behavior is the reverse of that found with metals. This so-called Joule effect in rubber, which is due to the latent heat of crystallization, may easily be demonstrated by stretching a rubber band and putting it to the cheek. It follows from thermodynamics that if rubber gets warmer on stretching, stretched rubber should contract when heated. Within a certain range of loads and elongations, this is found to be true, and stretched rubber has been used as a heat engine to transform heat into work.

The mechanical properties of rubber molecules and fiber crystals often are shown rather clearly by their effects on other gross physical or mechanical properties of the rubber, such as tear resistance. Tear tests are, in fact, useful in studying these structures, but they are complicated by a number of geometric factors.

If rubber remained an isotropic, elastic material when stretched, and the stress-strain curve of the rubber and the geometry of the sample were known, one could calculate the force needed to make a tear travel through a particular sample. If the sample has a straight edge it takes a considerable force to start a tear, but once started the force required to make it continue is surprisingly small. The reason for this is shown in Fig. 11, which shows the strain distribution at the edge of a thin piece of rubber that is being torn. The unstretched rubber was marked with two mutually perpendicular series of straight lines about 1/50″ apart. The first picture shows that when a piece of rubber having a straight edge is being torn, a considerable amount of rubber must be stretched to a high elongation before the edge is stretched to its breaking point. If a cut is first made in the rubber the strain distribution is that shown in the second picture. It is seen that only a very small area of rubber ahead of the cut is being stretched. Therefore, only a relatively small force is required to tear it.

As a rule, the calculated tear resistance differs widely from the observed tear resistance, but the discrepancies furnish clues to the structural changes which occur. The following example shows some of the complications that are observed. Samples 2″ wide and about 6″ long were stretched a definite amount, and while held at this elongation a cut was made across the strip



(a) (b)

FIG. 13. Effect of the order of stretching and cutting on the distribution of strains.

until a tear formed which traveled the remainder of the way of its own accord. This was repeated with different elongations, and the distance it was necessary to cut to start a tear was then plotted against initial elongation. Two points on this curve can be set down at once. At no stretch, the sample must be cut all the way across, and when stretched to the ultimate elongation, no cut is necessary to make it tear across. At first sight, one would expect a smoothly descending curve between these two points. Actually one may get a very different curve, as shown in Fig. 12, which was obtained with a high gum compound such as might be used in an inner tube. The curve rises at about 300 percent elongation, the same point at which one begins to notice a crystal structure in x-ray tests.

The explanation of these results is shown in Fig. 13. Sample *A* was cut $\frac{1}{4}''$ and then stretched so that the elongation ahead of the cut was about 500 percent. Sample *B* was first stretched to 500 percent, and then the cut was made. The lines on the samples formed squares in the unstretched state, so they indicate the strains in the stretched samples. The strain distribution in sample *A* is about that which would be expected in an isotropic elastic medium, but sample *B* has a definite rigidity. This rigidity causes the maximum at about 500 percent in the curve of Fig. 12.

An even more curious behavior is found if rectangular strips of tread stock are cut and then flexed through various cycles until the tears travel across the samples. The samples shown in Fig. 14 were all cut to the first line from the left edge and then stretched to different overall elongations. The results shown in Table I were

TABLE I.

| SAMPLE | STRETCHING CYCLE | LIFE (MIN.) |
|---|---|---|
| *A* | 0–60% | 10 |
| *B* | 10–60 | 25 |
| *C* | 15–60 | 90 |
| *D* | 17–60 | 150 |
| *E* | 20–60 | Infinite |

obtained. The life of sample *E* is considered infinite as far as this test is concerned, because the tear actually turned backwards. This phenomenon is related both to the molecular structure of the tread compound and to its abrasion resistance on the road.

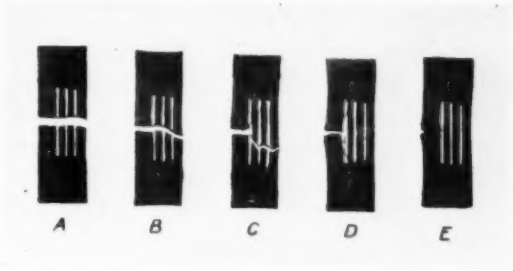



FIG. 14. Tearing of tread compound when flexed at various elongations.

### *IV. Product Development*

Nearly every problem in this field requires the closest cooperation of the engineer, chemist and physicist, and ultimately also the help of the production man and salesman, and usually it is impossible to evaluate the separate contributions of each group. This discussion will be limited to a few of the developments with which physicists have been concerned.

*Tires.*—The tires of an automobile have a number of functions: They furnish an air cushion under the car to take up road shocks; they transmit the driving and braking forces to the road; resist abrasion; and they make steering possible at the modern high speeds.

To fulfill the first of these functions, the tires must have an inner tube that will remain intact

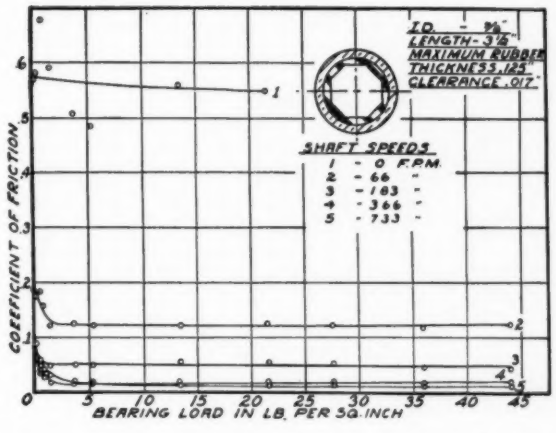



FIG. 15. Effect of load on coefficient of friction of a fluted rubber bearing.

mechanically over long periods of service, and also will prevent as much as possible of the air from diffusing through the rubber. The so-called puncture-proof tubes on the market use various means of keeping the tube intact, but there is not time to discuss them here. A number of methods of reducing the diffusion of air through the inner tubes are available, but they all increase the cost of the tubes, and with free air at every gas station, the public will not pay the necessary premium for a low permeability inner tube. The relative diffusion rates of different gases through rubber varies enormously, as shown in Table II.

TABLE II.

| GAS | RELATIVE RATE OF DIFFUSION | GAS | RELATIVE RATE OF DIFFUSION |
|---|---|---|---|
| Nitrogen | 0.16 | Carbon dioxide | 2.9 |
| Air | .22 | Ammonia | 8.0 |
| Oxygen | .45 | Methane | 13.9 |
| Helium | .65 | Water vapor | 50.0 |
| Hydrogen | 1.0 | Ethyl chloride | 200.0 |

Because of the difference in diffusion rates, air that remains in a tire for a year or so has considerably less oxygen and more nitrogen than normal air. The temperature coefficient of diffusion is surprisingly high, as shown in Table III. This explains why it is necessary to pump up tires so much more frequently in summer than in winter.

TABLE III. *Change in permeability with temperature for inner tube 0.054" thick inflated to 28 lbs./in.²*

| TEMPERATURE | | VOLUME OF AIR DIFFUSING OUT IN CU. IN. AT NTP PER 24 HR. | | |
|---|---|---|---|---|
| °C | °F | Per Sq. Ft. | From a 6.00–16 Tube | PERCENT OF TOTAL AIR DIFFUSING PER DAY |
| 0 | 32 | 1.4 | 13 | 0.3 |
| 30 | 86 | 9.8 | 88 | 2.0 |
| 60 | 140 | 24.5 | 220 | 5.6 |

The casing of the tire must withstand not only the stress due to the compressed air in the inner tube, but also all the driving stresses produced by the motor, and the stopping stresses produced by the brakes. Rubber has too high an elongation to do this satisfactorily alone, so it is reenforced by cotton cords and metal beads. Most of the tire development work to determine

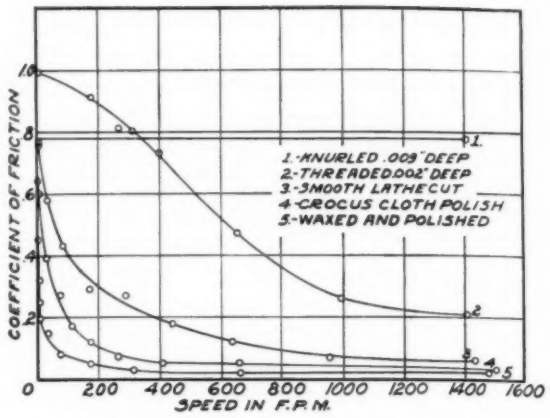



FIG. 16. Effect of speed and roughness of metal surface on the coefficient of friction against wet rubber.

the proper angle of the cords, the kind and size of breaker strips, the methods of tieing in the beads, and so on, has been on a cut-and-try basis. The tires were tested by running them on the road, or by running them on a Sprague dynamometer against a wheel having cleats on its surface. This type of development is costly,

FIG. 17. Test truck used to measure friction of tires on wet roads.

but it is the only type that can be used when there are so many unknown variables in the problem.

Attempts have been made to calculate the stresses in a casing but with only very limited success. The stresses in a thin-walled torus due to an internal pressure may be calculated, but when the torus is deflected by an external load,

as is the case when a car rests on the tire, the problem becomes more complicated. If, in addition, driving and braking stresses, and the effects of the finite wall thickness and the heterogeneous structure are considered, mathematical analysis of the problem is almost hopeless. But this does not mean that the mathematical attack is useless, for the solution of even the much idealized problem of the thin-walled torus furnishes a guide for designing tires of different sizes after the design of one size has been worked out empirically. This problem remains a challenge to our best mathematical talents.

FIG. 18. Some of tire designs cut from smooth treads.

Sometimes fundamental principles which are found in studying one product have a direct application in the development of entirely different products. Some years ago, a study was made of the lubrication of rubber bearings with water. The laws governing this system turned out to be quite different from those of metal bearings lubricated with oil, since the coefficient of friction is almost independent of load (Fig. 15). The coefficient of friction decreases with increasing speed (Fig. 16), the rate of decrease depending very much on the smoothness of the shaft surface.

This work helped improve the design of rubber bearings, but it apparently had little relation to the design of tires. However, when a tire skids on a smooth, wet pavement, it acts much like a cutless rubber bearing, so the principles that were used to make a rubber bearing with a low coefficient of friction could be used in reverse, as it were, to design a tread with a high coefficient of friction, or high skid resistance, on wet pavements.

The detailed development of these high friction tread designs required an enormous amount of work by tire engineers, as the designs had to be checked by measuring the coefficient of friction of full sized tires on the road. Equipment such as shown in Fig. 17 was used. The tire to be tested was put on one wheel of a trailer which was in line with the drawbar, which was attached, in turn, to a dynamometer on the truck. An observer on the truck operated an hydraulic brake on this wheel, and the force on the drawbar was automatically recorded. Tanks of water on the truck eliminated dependence on the weatherman for wet roads.

Some of the tread designs that had to be tested are shown in Figs. 18 and 19. The first group was made by cutting the designs in smooth tires, while each tire of the second group had to be made in a special experimental mold.

The curves in Fig. 16 show that in a rubber bearing the static friction is very high, and that it drops as the speed increases. The rolling tire is somewhat analogous to the static rubber bearing, while the skidding tire resembles the rubber bearing with the shaft in motion. One would then expect the coefficient of friction to be greater when the tire was just coming to a stop than it would be after it started skidding, and this was

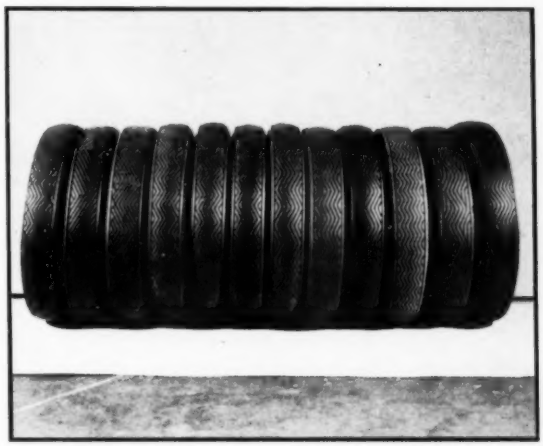


FIG. 19. Experimental molded tires used in tread friction study.

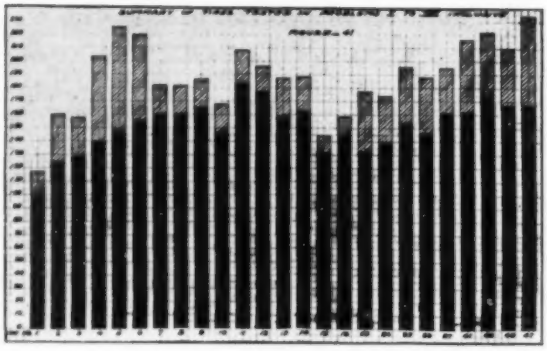


FIG. 20. Range of friction coefficients (ordinates) with different tread designs. Tire No. 1 at left has smooth tread.

found true in tests. Fig. 20 shows some of the results that were obtained when various tread designs were tested on a given wet road. The tops of the cross-hatched sections show the relative magnitudes of the maximum coefficients of friction obtained just before the wheels locked, or the impending skid resistance. The solid blocks show the relative frictions of the tires after skidding starts, or the full skid resistance. The block at the left, for the smooth tire, is taken as standard. It is seen that some designs give nearly twice the coefficient of friction or skid resistance of the smooth tire on wet roads.

The generation of heat in rubber by flexing caused a great deal of trouble in the old solid tires, and recently, due to higher operating speeds, it has become important both in fan belts and pneumatic tires. To measure the relative hysteresis of compounds more accurately than had been possible before, Dr. Lessig developed the Goodrich flexometer which is shown diagrammatically in Fig. 21. The cylindrical sample at $S$ rests on a lever system $B$, whose moment of inertia is increased by the two weights $W_1$. The weight $W_2$ puts the sample under a definite compression load while a high frequency vibration is

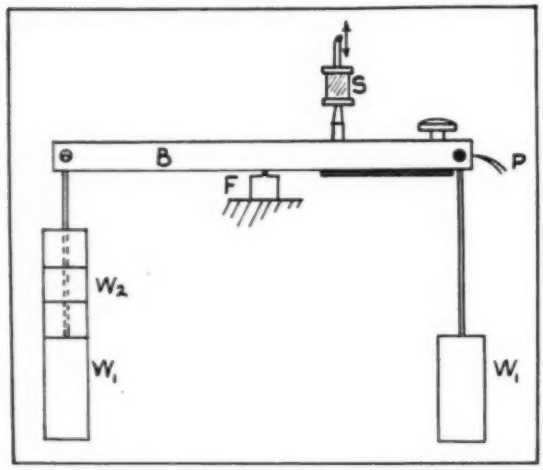



FIG. 21. Diagram of Goodrich flexometer for measuring hysteresis of rubber samples.

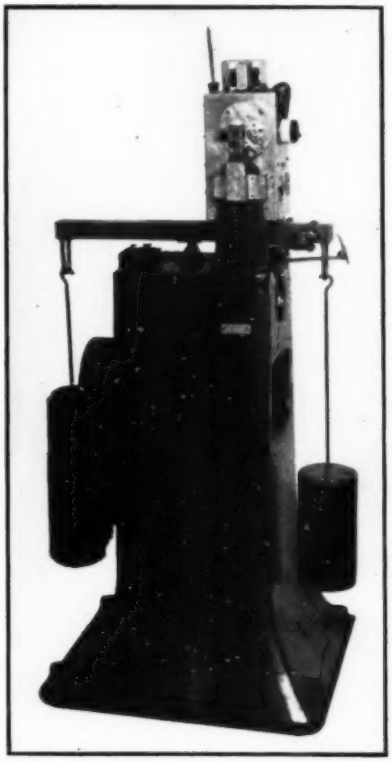


FIG. 22. Goodrich flexometer showing oven used in testing samples at high temperatures.

impressed on the sample through the upper anvil. The temperature rise at the base of the sample is measured by a fine wire thermocouple and potentiometer. Fig. 22 is a picture of the flexometer equipped with an oven for measuring the relative hysteresis of compounds at high temperatures.

*Motor mountings.* The use of rubber mountings to isolate the motor vibration in automobiles has led to a number of interesting problems and developments. One of the first problems was to

obtain compounds that had low permanent set. Chemists have greatly improved the compounds in this respect during the last few years.

Another important problem is to determine the shape of the rubber and metal parts to get the best combination of physical properties. Because of the high elongation of rubber and the low elongation of metal, purely geometric factors such as sample size and shape may produce some very curious effects. If a disk of rubber, say $\frac{1}{4}$" thick, is cured between two disks of metal, a sandwich-type mounting is formed. Shear strains of several hundred percent may then be applied to the rubber without injuring it. If, however, a direct tensile pull is applied to the rubber, some rather unexpected results are obtained.

FIG. 24. Internal failures of disk samples tested in tension showing cracks in rubber.

Figure 23 shows stress-strain tests for three mountings of this disk type which are 1", 3.75" and 5.75" in diameter, respectively, each having

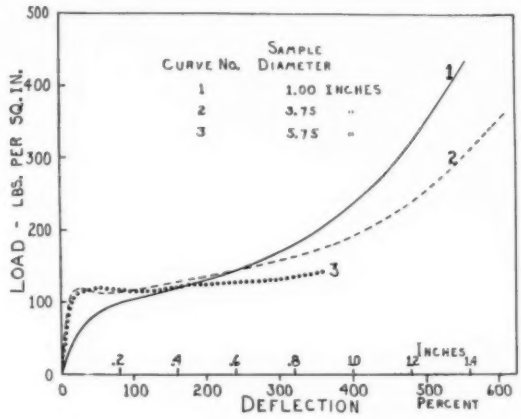
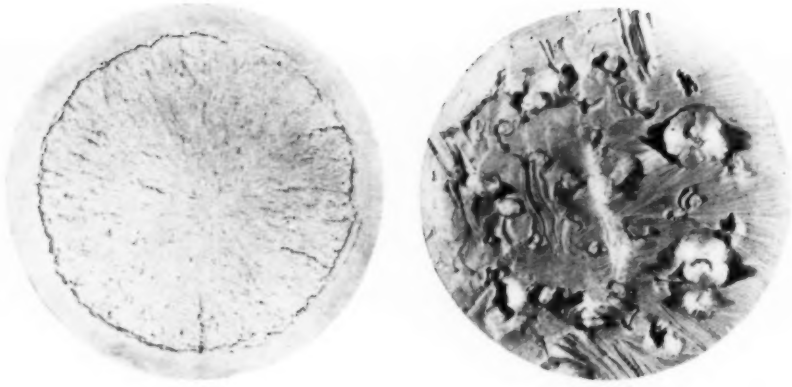



FIG. 23. Stress-strain curves of $\frac{1}{4}$ inch thick rubber disks cured to metal plates.

a $\frac{1}{4}$" layer of vulcanized rubber adjered to rigid metal disks. The first 25 percent elongation produces a marked increase in the load, but in the larger samples, at least, further increases in the elongation, up to several hundred percent, produce little or no increase in the load. Then, if the samples do not break, they stiffen up on further elongation. It should be noted that the 1" diameter sample shows this behavior to a much smaller degree.

These results are due to the fact that rubber is highly deformable, but it undergoes only slight change in volume when stretched or compressed, —that is, its Poisson's ratio is close to 0.5. When this disk sample is pulled, the rubber must "neck down" and pull in at the edge to keep its volume constant, since the area against the plates is fixed. A simple computation shows that the rubber at the edge is stretched to its ultimate elongation before the rubber at the center can be stretched very far. This produces large lateral tensile stresses on the rubber at the center, and these stresses produce cracks within the sample which may travel almost parallel to the initial tensile stress instead of at right angles to it, as one might expect. The cracks tend to relieve the lateral stresses and allow the sample to be stretched farther by the applied stress. The elongation may ultimately reach 400 or 500 percent before the samples fail by tearing normal to the direction of the applied tension. The pictures of the torn surfaces in Figs. 24 and 25 show some of the different kind of cracks that may occur.

These tests throw considerable light on the different mechanical structures present in these compounds, and they also have a bearing on specification tests of rubber mountings. Tensile tests of rubber mountings are justified as a check to make sure that the adhesion of the metal is above a certain minimum value, but it should not be assumed that a mounting that fails in a tensile



(a) (b)

FIG. 25. Internal failure of disk samples tested in tension showing porosity (a) and conical cracks (b).

test at 1200 lb. is 20 percent better than one which fails at 1000 lb., or that a mounting that has a dull, porous fracture is better than one with a shiny, conchoidal fracture. Compounders occasionally have had to sacrifice something in important properties such as creep in order to meet specifications for tensile strength, type of fracture or other properties which are not related to the behavior of rubber mountings in service. If a high tensile pull should really be needed in any service, it might be better to get it by changing the geometry of the mounting rather than by changing to a compound which exhibits a particular kind of fracture, and perhaps also has a higher creep.

The success of rubber motor mountings in isolating motor vibrations undoubtedly has stimulated the use of rubber mountings to isolate the vibration of heavy industrial machinery and street cars. In this field shear-type mountings are used almost exclusively, one model being shown in Fig. 26. The mills and plasticator shown in Figs. 1 and 2 are supported by this type of mounting. The mill and motor weigh about 45 tons, and the plasticator weighs 80 tons, all of this weight being supported by rubber in shear. The effectiveness of rubber mountings on these machines is shown in Fig. 27, the curve marked station 1–3 showing the building vibration produced by the plasticator when it was on a rigid mounting and the curve station 2–3 showing the vibration at the same place when the plasticator was supported on these shear-type rubber mountings.

The success of these large industrial and street-car mountings indicates that it is practical to replace the steel springs of automobiles with rubber. It has been pointed out that if used in tension, a rubber spring will store about seven times as much energy per unit volume, or about fifty times as much energy per unit weight as will steel. If the steel is put in the form of helical springs, and the rubber is used in shear as in street-car springs, the comparison at 4000 lb. load becomes, according to Hershfeld and Piron,

| | Rubber Spring | Steel Spring |
|---|---|---|
| Energy absorbed per cubic inch of space | 16.5 in. lb. | 27 in. lb. |
| Energy absorbed per cubic pound of material | 374.0 in. lb. | 237 in. lb. |

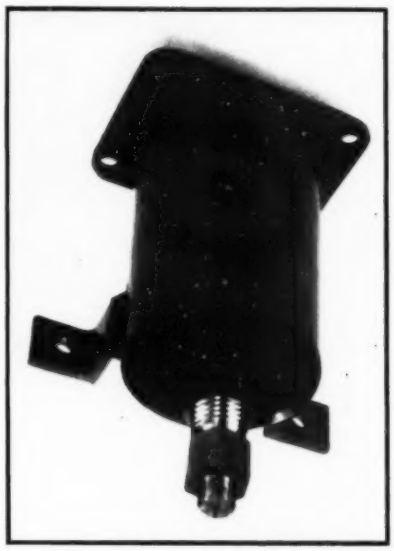


FIG. 26. Rubber spring used on mills and plasticator.

The problem of replacing steel springs with rubber in the auotomobile involves developing a mechanical construction which will magnify the distortions of the rubber so that it will carry high loads and yet have a very soft load-deflection curve. Among the various methods of accom-

plishing this, pneumatic devices having air under pressure held in flexible rubberized fabric bags have been suggested. A very simple method now being tried is to use rubber springs in torsion. These torsion springs can be "wound up" to support high loads, and yet have a soft load deflection curve. The small amount of creep or elastic after-effect that may develop after a few years service can easily be compensated by various means, such as by a slight increase in the initial twist given the bushings. This development may lead to a new market for rubber products and to simplified designs and improved riding qualities of the automobile.

### *V. Conclusions*

In this discussion only a few examples have been mentioned in which physics is being applied to increase our fundamental knowledge of the structure of rubber and the methods of processing and using it. The complexity of modern rubber products is so great that future improvements will depend more and more on this fundamental understanding of the structure of rubber and the relation of the structure to its gross physical properties.

Purely empirical testing will necessarily continue and will undoubtedly lead to important new discoveries. However, as the number of interdependent variables increases, the number of tests that must be made to cover the field *by empirical methods* increases overwhelmingly. Ten values of one variable can be studied in ten tests. With two variables 100 tests are needed and with ten variables, which is not unusual in rubber, $10^{10}$ tests are needed. At one hour per test, and working 24 hours per day, this would take something over a million years. Faced with a problem that grows at this rate, even the most practical man recognizes the limitiatons of purely empirical testing. If, through fundamental research, the relations between each of the ten variables were known, each variable could be studied independently and only 100 tests would be needed to cover the field. This goal may not be reached for a long time, but each step towards the goal will reduce by a considerable fraction the number of empirical tests that must be made. The potential contributions of physics to this fundamental study are being recognized more and more in the rubber industry.

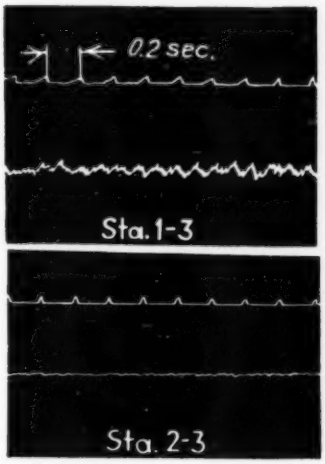



FIG. 27. Vibrograph record showing building vibration near plasticator when it was rigidly mounted (Sta. 1–3) and when mounted on rubber (Sta. 2–3).

This fundamental and applied research being carried on with the cooperation of all groups of technical men—engineers, chemists and physicists—will result in new and still better rubber products that will not only keep pace with, but, we hope, will also help to make possible, the new and better automobiles that are coming in the future.

# Some Physical Problems in Noise Measurement*

BY PAUL HUBER

*General Motors Proving Ground, Milford, Michigan*

TO the automobile industry, noise measurement is simply a method of obtaining information which will enable those in charge of car design to produce a more comfortable vehicle. To the engineer assigned to the work, there is a double problem; to obtain clear, reliable objective data, and to interpret it. In the course of our work at the General Motors Proving Ground we have encountered both problems, and will outline here some of the physical factors involved. Detailed interpretation of test results and design modification must remain the competitive field of the different groups.

Total noise meters have been available for some years and their value in noise problems cannot be questioned. They are particularly worth while in rating and comparison tests where the noise spectrum is always the same. We have been using them both for intercomparison of all makes and models of automobiles and for repeat tests on single cars. Table I gives an idea of the reproducibility of measurement of road rumble, with the microphone in a single position in the front seat of a car. On such tests, the noise pattern in the car rarely makes 2 db difference in the neighborhood of the correct position, although 0.5 db is often found. In this case the average deviation of a single observer on the repetition of a single test is only 0.25 db, but the average of the readings on the two meters on car B disagree by 1.2 db. Since this test was run only to check the consistency of the observers, this is probably due to careless placement of the microphones on the seat of the cars, but may be due to a different response characteristic of the output meter. The noise fluctuation is entirely different in these two cars and this fluctuation will cause different average readings unless the two meters are identical in speed of response and damping. The disagreement is not due to the relative response of the two meters. On that day the over-all response curves including microphones were parallel, with No. 134 about 0.4 db lower than No. 129.

The shape of the relative frequency response curve of the noise meter on the 70 db weighing net is a great source of indeterminacy in comparisons between different cars. The effective frequency of a car is usually below 100 cycles, where the curve is quite steep. Two cars which have the same intensity at different effective frequencies may give markedly different readings on meters with non-uniform response. The use of a flat response meter does not greatly help the situation, for such a meter will place cars farther from the order in which they are judged by the ear than a meter with suitable frequency response. The only solution seems to be the use of a compromise response, with frequent recourse to sound jury methods for

TABLE I. *Noise level on Belgian block road. Repeat test—30 m.p.h.*

| Meter | | Car A | | | | Car B | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 129 | | 134 | | 129 | | 134 | |
| Observer | Driver | db | δ | db | δ | db | δ | db | δ |
| 1 | 2 | 92.4 | | 92.7 | | 89.7 | | 88.6 | |
| | | 92.6 | 0.2 | 92.6 | 0.1 | 89.5 | 0.2 | 88.6 | 0 |
| 2 | 4 | 93.0 | | 92.3 | | 89.1 | | 88.0 | |
| | | 92.7 | .3 | 92.7 | .4 | 89.8 | .7 | 88.0 | 0 |
| 3 | 1 | 92.7 | | 92.6 | | 89.6 | | 87.8 | |
| | | 92.8 | .1 | 92.9 | .3 | 89.5 | .1 | 88.3 | .5 |
| 4 | 3 | 93.0 | | 92.8 | | 89.6 | | 88.6 | |
| | | 93.1 | .1 | 93.1 | .3 | 89.5 | .1 | 88.3 | .3 |
| Average | | 92.8 | .2 | 92.7 | .3 | 89.5 | .3 | 88.3 | .2 |

* Paper presented at the Symposium on Physics in the Automotive Industry at Ann Arbor, Michigan, March 14 and 15, 1938.

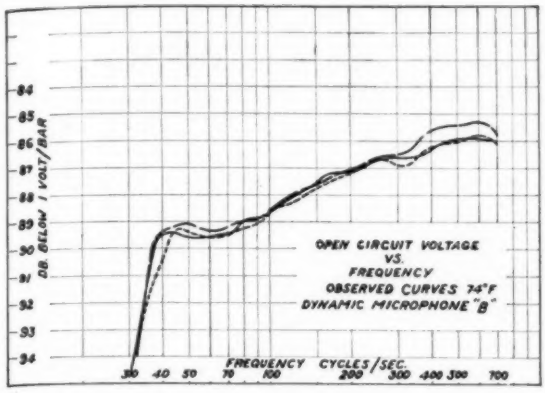



FIG. 1.

decisions on "bad" and "good." Repeat tests on one car are in better agreement with the ear, if the changes made do not greatly modify the noise spectrum.

Since we know that by careful work observers can check readings to a few tenths of a decibel, the question arises as to the accuracy of the noise meters over a long period of time. Our experience leads us to believe that the problem of reliable electron tube amplifiers has been solved, if a suitable means of checking over-all gain is used. The internal calibration means now included in commercial noise meters is not satisfactory. A common type uses the output meter of the noise meter to check the voltage of an internal oscillator or of the power line. A fraction of this voltage is then fed into the amplifier. Since the copper oxide rectifier output meter is temperature sensitive, the result is that the response of the whole meter is temperature sensitive. Our meters are in

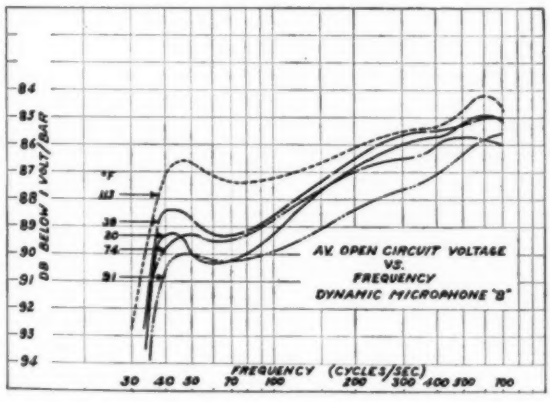



FIG. 2.

error by 0.03 to 0.05 db per degree Fahrenheit when the internal calibration is used. We are now using an external oscillator and a meter free from temperature error for routine calibration, but the method is quite inconvenient. One other warning should be given. By far the most irritating source of error is that due to cords and connections. Only in rare cases can we use a microphone mounted directly on the meter, so extensions are a necessary evil. Defective cords, plugs and jacks cause more random error than the operator and may cause continuous error extending over days. The best routine check we know is to use an electrical calibration in series with the microphone. The common parallel or substitution methods will not ordinarily show open circuits in cords.

A major source of uncontrollable error seems to be in the microphone itself. We have found that

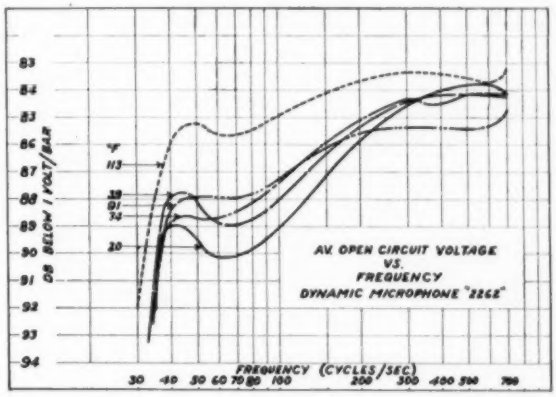



FIG. 3.

the moving coil microphones are best suited to our work. They are extremely rugged, retaining their calibration in spite of rough handling and occasional careless use. Their worst faults are an irregular temperature coefficient and magnetic pick-up. The temperature effects will be discussed later. Crystal microphones have a wide field of application but are less adapted to automotive problems. Their temperature coefficient is greater than that of the moving coil microphone, and at elevated temperature they first lose their low frequency response and then become inoperative. The crystals also seem less rugged. When used on an extension cord, any high impedence unit, such as a crystal microphone, is subject to pick-up of electric fields. Our experience is that this results

in a greater pick-up from the ignition circuits of an automobile than is found in the low impedence microphones. A second source of trouble, due to its high impedence, is the effect of moisture on extension cords. As far as we can tell, the crystal microphone itself is free from humidity error, but we have found cases where the low frequency response has been lost by leakage in the cord. The use of the condenser microphone for routine automotive work is limited by the difficulty of using it on an extension cord, except at the expense of fragility or poor low frequency response, and humidity error.

We have been forced to study the temperature response of microphones in the low frequency region by the wide temperature range we en-

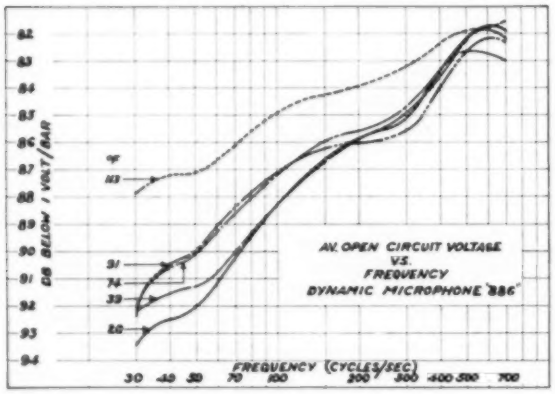



FIG. 4.

counter. In the winter it is not uncommon for the interior of cars without heaters to stabilize below freezing. In the summer, the air temperature with the windows closed may exceed 110°, and we have found temperatures above 120° in engine dynamometer work. An electrostatic actuator was built to give a pressure calibration of a condenser microphone at any temperature in this range. This microphone was then used as a transfer standard to obtain an acoustic calibration of other microphones at that temperature. The method is trustworthy between 30 and 700 c.p.s., which covers the essential working range. Fig. 1 shows three curves taken on microphone *B* at room temperature on three different days. The deviation of these curves from the average is less than 0.5 db. The absolute calibration of the actuator is in doubt by 1 or 2 db, because of the

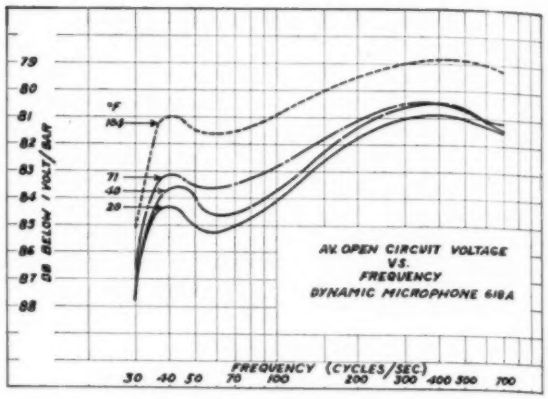



FIG. 5.

difficulty of exactly determining the spacing and voltages.

Figures 2, 3 and 4 show average open circuit output voltage curves for Western Electric 630A microphones *B*, 2262 and 886. These figures form a group, in that the data were taken in sequence on the three microphones. That is, a curve was observed on each one of the temperatures. This set of data was then taken twice again and all readings averaged. The reason for this explanation is that the 91° curve seems to be out of order in the curves for microphones *B* and 2262. We cannot ascribe this to random error unless we are also willing to change the same curve for microphone 886. It is obvious from these curves that the temperature errors are greatest at low frequencies, and that the rate of change is greater above room temperatures than below. By chance, microphone 886 was run with the acoustic weighing net, which raises the response at the very low

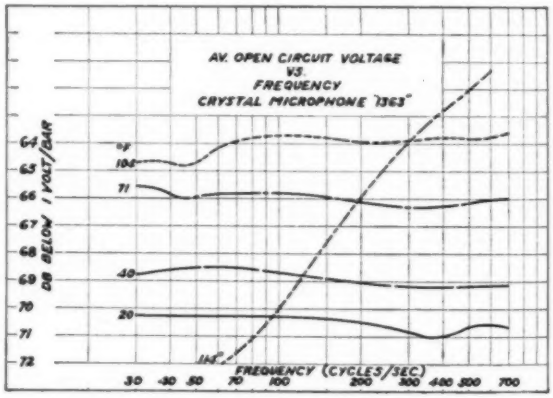



FIG. 6.

frequencies, blocked off. This accounts for the difference in the shape of the curves on Fig. 4. Fig. 5 shows a similar set of curves for a W.E. 618A microphone, No. 1581. The temperature

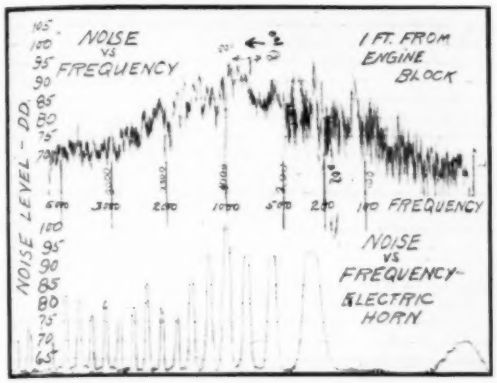



FIG. 7. Original record of the relation of noise level *vs.* frequency.

effect is about the same in magnitude and character as that of the 630A microphones. It was run at the same time as the Brush BR2S, No. 1365 crystal microphone, Fig. 6. The response curves of the crystal microphone are of a different type. With the exception of the curves for 104° and 114°, they are a better check on the accuracy of the method than of the frequency response of the microphone. At high temperatures, the output of the microphone falls at low frequency. A hint of this is shown in the 104° curve, and a marked effect in the 114° curve. The cause is a reduction in the d.c. resistance of the crystal with temperature.

There is both pleasure and sorrow in the study of these curves. Persons who do their work over a narrow range of temperature will find assurance that their errors are negligible, although our reaction is that it makes small differences in cars unmeasurable if the temperature changes over the upper part of our normal working range.

### *Interpretation*

The second field referred to in the introduction does not always require the accuracy we have been discussing. Its need, rather, is more complete information than can be obtained with a total meter. Frequency analysis has been described several times in various journals and all we can do here is to emphasize its value and, in particular, to emphasize the value of the combination of frequency analysis and recording. Much work has been done by the use of band pass filters with a total meter and there are problems which are best attacked this way. The use of a tuned heterodyne analyzer is nearly always preferable to filters on machinery noises, because the major offenders are usually discrete components definitely related to the speed or structure of the machine. In addition to these components, there is a background of irregularly varying noise which is usually termed "unpitched." A point by point survey of the noise spectrum will yield much information but requires a great deal of time, for the output of the

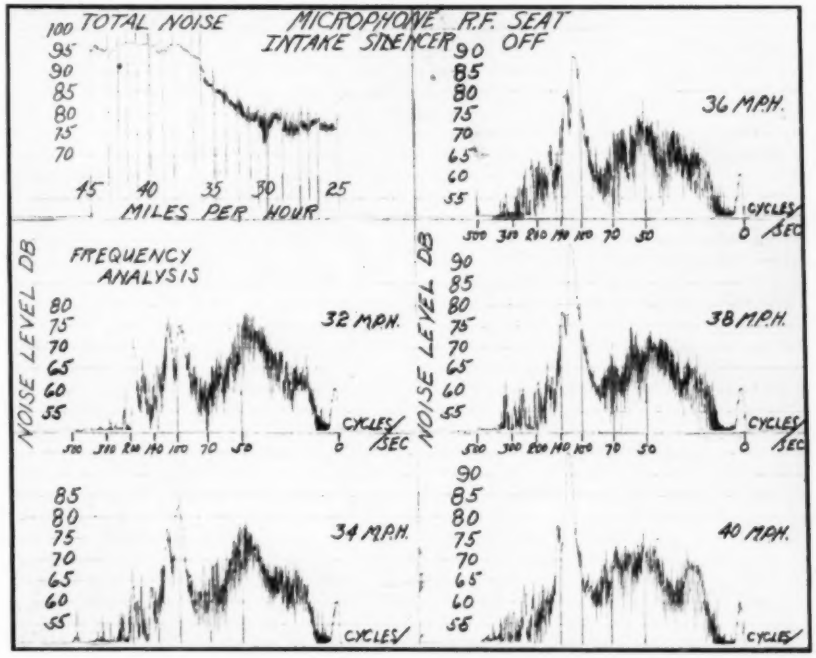



FIG. 8. Original records of noise levels *vs.* frequencies.

analyzer fluctuates badly except when it is on a component several decibels above the background. Accordingly, time must be taken to observe each large deflection of the meter to determine the average intensity of the peak. A recorder operating on the output of the analyzer as it is continuously swept through the frequency range gives a record showing the rapidly fluctuating continuous background with fairly sharply defined peaks on it. Such a record observes each component for only a second or two, so the intensity recorded may be the maximum or minimum rather than the average. In effect, the recorder enables an operator to greatly increase the quantity of data which may be obtained in a given time, at some expense of accuracy.

Figure 7 shows two records of frequency sweeps taken of widely different mechanical noises. The lower one was taken on an automobile horn of the air tone type. There is practically no background and the components are closely harmonic. The upper record was taken on an experimental engine under conditions of very rough combustion. The unusual intensity of frequencies above 400 c.p.s. is obvious to anyone who has made analyses of engine noise.

For another example of frequency-intensity recording we have chosen an intake noise period. Fig. 8 shows records of the noise in the right front seat of a car at full throttle, at various speeds. The curve at the upper left presents a total noise intensity *vs.* speed record, taken while accelerating, showing a marked increase near 36 m.p.h. Analyses at the steady speeds indicated are given in the other records. The sudden increase in intensity of the component near 120 c.p.s. is easily noted. This type of period is not typical of present production automobiles, for such intake periods have been eliminated by intake silencers for some years. The records were taken without a silencer in order to give an example which is common to all automotive engines. However, this method of attack on period noise may be used on any machinery noise which is not constant with speed and load. We frequently find that a change in quality of noise may result in an objectionable period without measurable change in total intensity. In such cases, only frequency analysis will be worth while and the use of a recorder generally simplifies the problem of obtaining the information.

Another class of problems to which this technique can be applied is that of mechanical vibration. In the automotive field, vibration is almost always associated with noise production, except for a few very low frequency vibrations, such as those which occur in the spring suspension. Vibration measurement at frequencies above 30 c.p.s. requires only a suitable transducer to change vibration energy to electrical energy to enable all the technique of noise measurement to be used. The various types on the market have their own particular application and their own types of error. No effort will be made here to cover the subject. The use of an artificial vibration source as an aid in obtaining engineering information is often very desirable

The use of frequency analyzers, recorders and vibrators is subject to the ingenuity of the operator. Special tests for each job, plus changes to the car to prove or disprove tentative hypotheses are usually required before a solution is finally reached, for the mechanism of noise production and propagation in machines is not yet understood. We are certain that in most cases the cheap and effective way to eliminate noise is to do it in the major design of the machine, not by trying to patch it up as a last resort. Such design changes can only be made as a part of the compromises which produce modern automobiles. The acoustical engineer or the physicist cannot ask for design changes unless he understand the mechanism of noise production so well that he can adapt his desired change to the other requirements of design. When this degree of understanding is reached, a cure can usually be effected.

# Photoelastic Analysis Practically Applied to Design Problems*

BY O. J. HORGER

*Timken Roller Bearing Company, Canton, Ohio*

## *Introduction*

PRINCIPLES of optics combined with the theory of elasticity provide an experimental basis for studying stresses in structural and machine design parts, known as the photoelastic method (1–4).[1, 2] This method has an important practical advantage in that it provides the engineer with a photograph showing the distribution of stresses and their magnitude and direction in the design part under consideration. Many problems are continuously confronting the designer where no satisfactory analytical method of calculating the stress is available. Frequently the shape of the design part as well as the distribution of load acting presents serious difficulties in calculation and photoelasticity is useful in providing a practical solution.

The trend in industry toward reduced weights and higher speeds requires a better knowledge of the actual stresses present in order to reduce the factor of safety. In this respect photoelasticity is useful in locating local regions of stress concentration and indicating alterations in the design to give more uniform stress distribution throughout the member as a means of obtaining maximum strength at minimum cost.

So far, photoelasticity has been limited to the study of stresses in two dimensions, that is, where the loads are acting in the plane of the design part. Many problems are in three dimensions and in order to make a photoelastic study it becomes necessary to introduce some modifications so as to reduce the problem to a two-dimensional one (5).[3] In such cases the results obtained are generally of a qualitative nature so that they afford the engineer a better understanding of the problem under consideration and suggest what improvements may be made. Little progress has been made toward three dimensional stress anal-

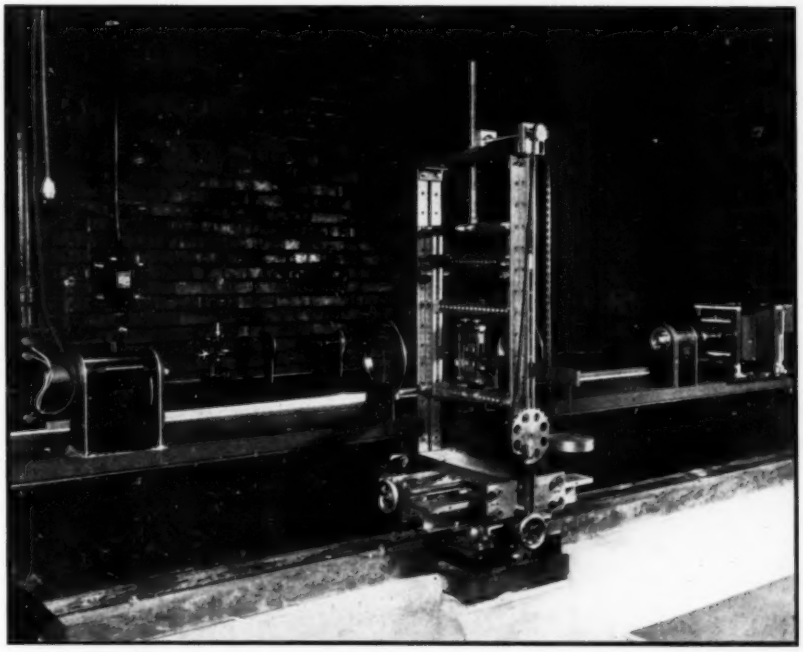


FIG. 1. Photoelastic equipment.

* Paper presented at the Symposium on Physics in the Automotive Industry at Ann Arbor, Michigan, March 14 and 15, 1938.

[1] The figures appearing in parenthesis refer to bibliography at the end of this paper.

[2] Treatise on photoelasticity may be found in reference (1) while (2, 3, 4) give detailed information on this subject.

[3] Good agreement is shown in (5) between photoelastic tests in two dimensions with the three-dimensional case on axle fillets when strain gauge measurements were made.

ysis by photoelastic methods but various investigators are continuing the research in this field (6).

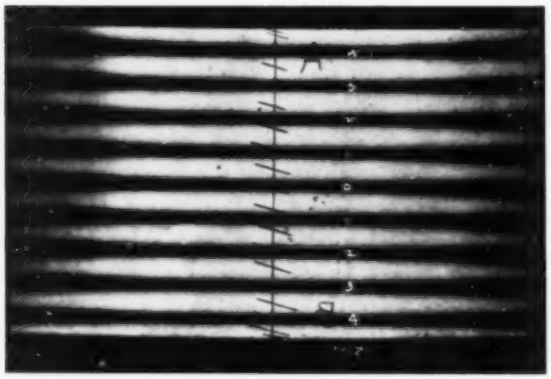


FIG. 2. Fringe photograph of beam in uniform bending.

### *Photoelastic Equipment*

The optical equipment and set-up used to make photoelastic studies in the research laboratories of the Timken Roller Bearing Company is shown in the photograph of Fig. 1. This entire set-up was built to our specifications, requiring fixed mounting longitudinally of all optical parts as a means of maintaining accurate alignment and avoiding the necessity for making adjustments. The installation is divided into three sections, the polarizing stage, loading frame, and analyzing stage.

The polarizing stage located in front of the loading frame includes, from left to right, a light source, condensing lens, water cell, turret head containing various filters, 15 mm square Glan Thompson prism mounted in 360° graduated and rotatable mount, and quarter wave plates in graduated and rotatable mount which is hinged so that it may readily be swung in or out of the optical path. A 7″ diameter field lens of a plane convex type is shown adjoining the loading frame. High intensity mercury arc light source is shown but a 15-ampere carbon arc lamp is also available to interchange with the mercury light.

The loading frame is adjustable in three directions and is designed to receive various types of models for study. Provisions are made for applying various types of loads to the models. The model shown in position is the one discussed later in this paper under press fits.

Analyzing stage includes the 7″ diameter field lens which is an achromatic doublet to remove color and distortion. This is followed by a single housing in which is mounted quarter-wave plates and analyzer prism which are duplicates of those in the first stage, a 4″ lens, and finally a revolving mount containing a 3″ lens for unity magnification of the object and a micro-lens for four times magnification. A standard 8″×10″ view camera is shown for taking a photograph of the image.

### *Principles of Photoelasticity*

A model of the design part to be studied is machined from a flat plate of some isotropic transparent material, usually Bakelite because of its commercial availability and high optical coefficient and other desirable characteristics. Other materials may be used such as glass, celluloid or Marblette (7). Such materials under stress have the property of double refraction which is the fundamental basis underlying the optical principles employed in photoelasticity. As early as 1816, Brewster discovered this property and recognized its engineering value but little use was made of this knowledge until Wilson in 1891 studied beams and Mesnager investigated bridge stresses in 1901.

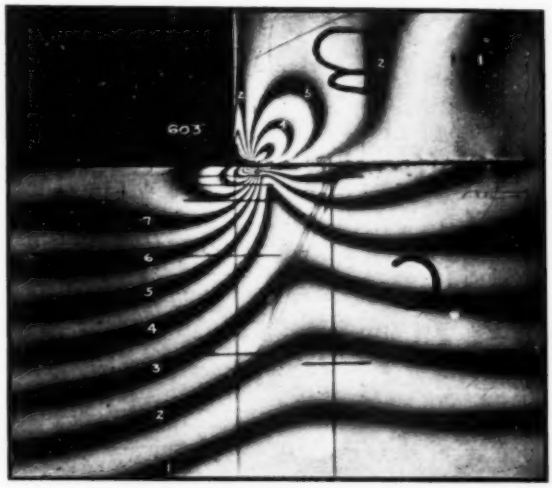


FIG. 3. Fringe photograph of usual press fit assembly. Portion shown above the horizontal center line of the axle and one-half of the hub length.

The lack of a suitable material was largely responsible for the little practical engineering application of photoelasticity. The use of glass by the early investigators for models presented difficulties in cutting the models as well as obtaining sufficient optical effect.[4] The development of plastic materials during and following the world war gave impetus to contributions of a useful engineering nature by various investigators such as those by Coker and Filon (1, 2, 3).

The substitution of Bakelite in the model for the actual material used in the machine parts often leaves some question with the engineer as to the validity of the results. This feeling is not justified, however, when we consider that the stress distribution in simply connected bodies remains the same regardless of the elastic constants for stresses below the elastic limit.

The model is loaded in a manner similar to that of the actual member. When polarized light is passed through the stressed model each element acts as a double refracting crystal so as to break the imposed beam into two emergent rays.[5] These two rays, while passing through the specimen, travel at different velocities so that for most materials the velocity of a ray vibrating in a plane of compressive stress is greater than that of a ray vibrating in a plane of tensile stress. Thus there results a certain relative displacement between the two rays at each point of the specimen. These rays are passed through an analyzer which selects the components of the rays vibrating parallel to its principal planes. The amount of out-of-phase or relative retardation of the two final emergent rays is proportional to the difference of the principal stresses and results in an interference effect to form the projected image. This image is a series of colored bands when white light plane polarized is used or with circularly polarized monochromatic light a series of black fringes is produced. For each wave-length of retardation an additional fringe appears. This paper reproduces a number of photoelastic studies using black fringes as the colored bands are both difficult to reproduce and lose their identity after going beyond several spectrums.

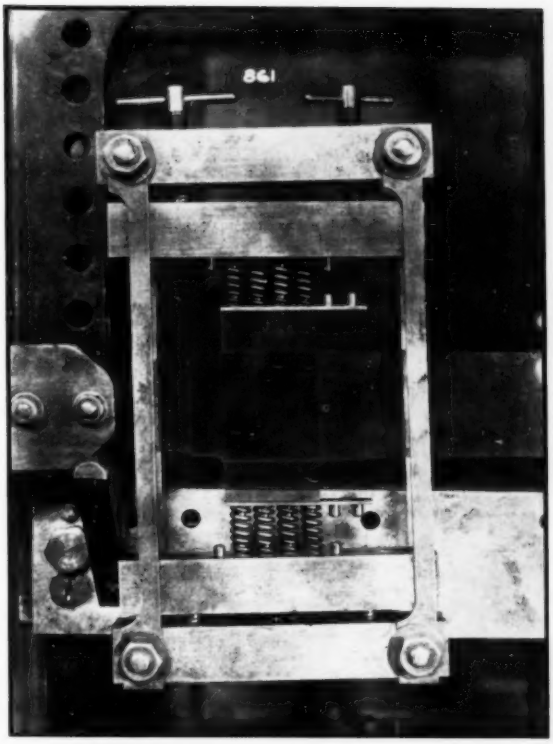



FIG. 5. Set-up Bakelite model for press fit study.

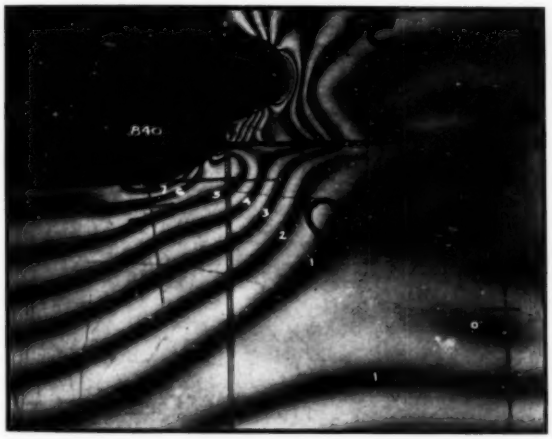



FIG. 4. Fringe photograph of improved press fit assembly. Portion shown above the horizontal center line of the axle and one-half of the hub length.

### *Interpretation of Photoelastic Photographs*

A typical photograph showing these black fringes is shown in Fig. 2, for the simple case of a beam subjected to a uniform bending moment.

[4] Optical sensitivity of Bakelite is about 20 times that of glass and about 4 times that of celluloid. Marblette has higher optical coefficient than Bakelite.

[5] The optics of this method is well detailed in references (2, 4).

The black fringes represent lines of equal maximum shearing stress. It will be noted that the fringes are straight, parallel, and uniformly spaced. The fringes are numbered with the neutral axis marked zero. On each side of the neutral axis the fringes are numbered 1, 2, 3, etc., up to the boundary of the beam. The shearing stress is directly proportional to the fringe number. This example conforms with our elementary theory of bending stresses in beams where the stress at any point is calculated as being proportional to the distance of the point from the neutral axis.

The photoelastic photographs referred to later in this paper will show regions of fringes which have sharp curvatures and close nonuniform spacing and this is an indication of stress concentration. Such regions are a source of weakness in the design of any member and the desired fringe pattern is one of uniform stress similar to Fig. 2, which distribution we know gives maximum strength in bending. One of the objects of making photoelastic tests is to determine the magnitude of these maximum local stresses and then change the design to reduce this effect of stress concentration to obtain greater strength.

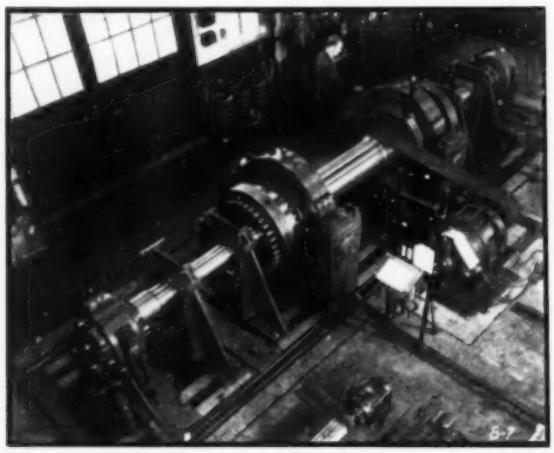


FIG. 6. Fatigue machine shown testing $11\frac{1}{2}$" diam. axles.

So far only shearing stresses have been mentioned as being ascertained from the fringe photographs. When the fringes appear on a free boundary as at the edge of the beam, Fig. 2, or at a hole, Fig. 11, or fillet, Fig. 13, then the magnitude of one of the principal stresses become zero and the other principal stress becomes known from the fringe order at the boundary.

FIG. 7. Typical fatigue failure of axle test in Fig. 6. Axle shown with wheel removed.

In fatigue problems the maximum shearing stress is often used as a criterion of failure, especially for ductile materials. Therefore the fringe photograph gives directly this value of maximum shearing stress and it is not necessary to determine the value of the principal stresses and their direction throughout the model, which investigation is more difficult to make and is not shown here (8).[6]

### *Practical Application of Photoelasticity*

The designer must provide sufficient strength to withstand the imposed forces whether they result from static, fatigue, or impact loading. Under the first case of static loading the values of stress concentration derived from photoelastic studies may be directly applied but in the latter two cases some consideration must be given to the factors of shape and size of the member and the "sensitivity" of the material to stress concentration. In the case of fatigue tests of small size specimens of the actual material being used it has been found that the fatigue stress concentration factor is usually less for ductile materials than is found from photoelastic studies (5). When the design members are large or the "sensitivity" of material is high it is often considered that the actual stress concentration factor approaches the photoelastic value as a limit and this procedure, in the absence of more detailed data, represents good design practice. Photoelasticity gives the stress concentration due to shape and the influence of other conditions such as corrosion cannot be ascertained from this method. In any

[6] The bibliography of paper in (8) contains references to this subject.

FIG. 8. Fringe photograph of contact stresses.

case only stresses within the elastic limit may be considered.

For the above reasons photoelastic tests are often used to correlate and supplement tests of actual design parts. Some examples of design problems given in this paper show this correlation. A small number of some of the most important of many problems developed by various investigators is briefly summarized as follows:

### *Press and Shrink Fitted Assemblies*

Press fitted assemblies such as railroad wheels, propellors, gears, pulleys, wheels, and bearings on axles or shafts and other types of press fitted construction elements are very frequent in engineering practice. Seldom is it realized, however, that due to the press or shrink fit the fatigue strength of the axle is reduced to about $\frac{1}{2}$ to $\frac{1}{6}$th of that of the same axle without the fitted member. This tremendous weakening effect was investigated by both photoelastic methods (9, 10) [7] and fatigue tests (10–15).

[7] Also see discussion to paper (9) in J. App. Mech. 2, A-71 (1935).

Figures 3 and 4 give the results of photoelastic studies made by the author using the model set-up shown in Fig. 5. Fig. 3 shows the stress condition for conventional design practice followed in press fitted assemblies. It will be noted how the fringes loop together and funnel toward the end of the wheel fit. Failures of axles in service occur in this region of localized high stress. Fig. 4 shows how this condition may be improved by mounting the wheel on a raised seat of the axle and providing an annular relief groove in the hub face of the wheel. This improvement is indicated by the comparatively less number of fringes and by the longer curvature and increased distance between the fringes. The usual press fitted assembly produces a shape condition similar to that obtained if the wheel hub and axle were machined from one piece with a very small fillet at the junction between the axle and the end face of the wheel. We all have a physical feeling for the weakness of such a sharp fillet and in this respect the improved construction in Fig. 4 gives the effect of using a much larger fillet.

This press fit problem is one combining (a) stress concentration factor due to shape plus (b) an added weakening effect due to rubbing corrosion (10).[7] The photoelastic test gives the value of the factor due to (a) and actual fatigue tests are being made to determine that due to (a) plus (b). These fatigue tests are being made on full size railroad axles $11\frac{1}{2}''$ diameter having a press fitted wheel. A photograph of this machine and axles being tested is given in Fig. 6 and an axle fatigue failure developed in this test is shown in Fig. 7. It should be observed that the axle in

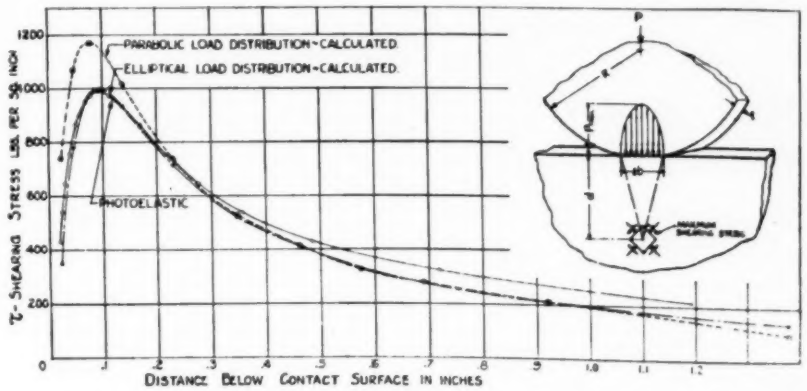



FIG. 9. Curves of maximum shearing stress at various depths below contact surface.

Fig. 7 developed a fatigue fracture within the wheel fit a short distance from the hub face, at 10,500 lb. per sq. in. nominally calculated bending stress. Small diameter plain specimens cut from these large axles and tested without a press fit gave an endurance limit of 55,000 lb. per sq. in. From this example, the weakening influence of the press fit is about 5 to 1 and this is a problem upon which much further research in fatigue tests is being made in order to improve the axle strength.

### *Contact Stresses*

The problem of contact stresses is associated with gear teeth, cams, rollers, and wherever two bodies come into contact under pressure. This particular photoelastic study was made to determine if the pressure across the width of contact between two diametrically loaded parallel cylinders was distributed according to an ellipse (16) or a parabola (17). A typical fringe photograph is given in Fig. 8 from which an analysis may be made similar to that shown in the curves of Fig. 9 indicating the shearing stresses along the vertical center line of contact at various distances below the surface. Fig. 9 indicates that this pressure is distributed according to an ellipse across the width of contact $2b$.

The practical importance of knowing this distribution is principally for determining the magnitude of the maximum shearing stress which occurs below the surface. Failures in ductile materials are sometimes predicated on the basis of maximum shear theory and in such cases failures would occur below the surface. Thin platings applied to parts subjected to contact stresses may peel off if the thickness of the plating is about the same as the depth of maximum shear stress below the surface. Rollers of a ductile material were in-

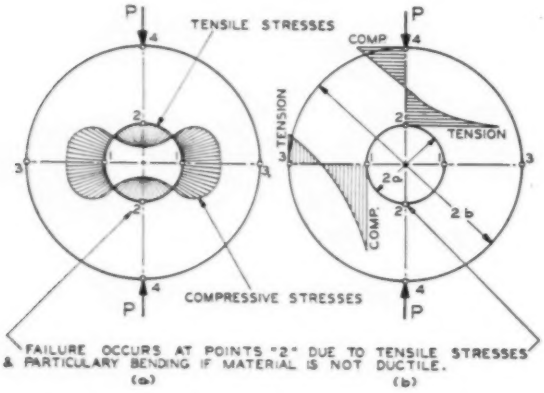



FIG. 10. (*a*) General system of tangential stresses around hole. (*b*) Stress at critical sections max. value of stresses at points 1, 2, 3, and 4. May be calculated from curves in Fig. 12.

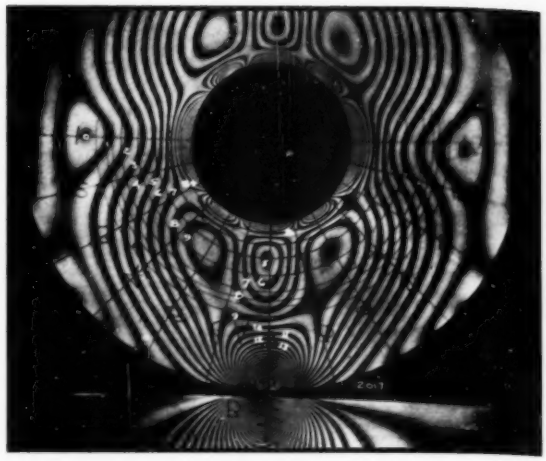


FIG. 11. Fringe photograph of hollow cylinder.

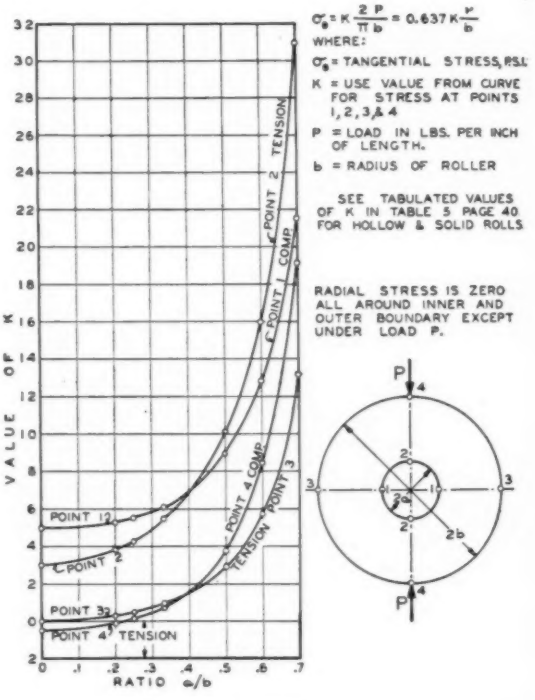

$\sigma_\theta = K\frac{2P}{\pi b} = 0.637K\frac{P}{b}$
WHERE:
$\sigma_\theta$ = TANGENTIAL STRESS, PSI.
K = USE VALUE FROM CURVE FOR STRESS AT POINTS 1, 2, 3, & 4
P = LOAD IN LBS. PER INCH OF LENGTH.
b = RADIUS OF ROLLER
SEE TABULATED VALUES OF K IN TABLE 5 PAGE 40 FOR HOLLOW & SOLID ROLLS
RADIAL STRESS IS ZERO ALL AROUND INNER AND OUTER BOUNDARY EXCEPT UNDER LOAD P.
VALUE OF K
RATIO a/b
POINT 1 TENSION
POINT 2 COMP.
POINT 3 TENSION
POINT 4 COMP.
POINT 1 COMP.
POINT 2
POINT 3
POINT 4 TENSION
2a
2b

FIG. 12. Value of $K$ for calculation of critical stresses in rollers for various values $a/b$.

vestigated (18) and found to fail below the surface at this point of maximum shearing stress.

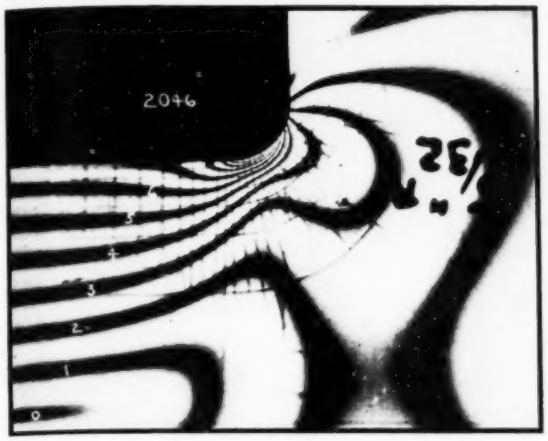



FIG. 13. Fringe photograph at fillet.

### *Hollow Cylinders*

A number of applications are found in practice where hollow cylinders are subjected to diametrical loading shown in Fig. 10. The problem is to determine what maximum stresses are developed at the boundary of the hole for various proportions of $a/b$.[8] Photoelastic studies were made of various Bakelite models with various $a/b$ ratios and a typical fringe photograph is shown in Fig. 11. Summary analysis of a number of such fringe photographs is expressed in Fig. 12 which enables the designer to calculate the critical stresses and predict failure.

### *Fillets, Holes and Notches*

These are common forms of stress concentration with which every designer is familiar and in these cases photoelasticity has furnished a very complete analysis and solution for practical engineering purposes.

A typical fringe photograph is shown in Fig. 13 for a fillet in bending. A number of such tests on various fillet ratios have been made by the author but more complete analysis of this fillet problem has been made by Weibel (20) and Frocht (21). Summary curves of practical use to the designers are quoted from Frocht in Fig. 14.

[8] This problem is calculated when $a/b$ is $\frac{1}{2}$ in reference (16) and when $a/b$ is $\frac{1}{3}$ in reference (19).

To determine the maximum stress at the fillet the nominally calculated bending stress at the fillet is multiplied by the stress concentration factor $k$ given on the curves of Fig. 14. Interesting work on constant stress fillets has been done by Baud (22).

The problem of notches and holes is very completely covered by curves similar to Fig. 14 which are available in the literature (21, 23, 24).

### *Bolts*

Tests have been made on the screw threads of bolts but reproductions of photoelastic studies (1) are not given here. They show that the first few threads carry most of the load as evident from actual failures in service. The usual bolt and nut standards most frequently used do not give maximum fatigue strength and where heavy duty applications of bolts are concerned it becomes necessary to vary from such standard practices.

In this connection it is interesting to note Fig. 15 which is quoted from Wiegand (25). The results of actual fatigue tests shown indicate that considerable improvement may be made with little additional cost. In most cases this improvement is due to distributing the load over a greater number of threads and thereby reducing the magnitude of the highly localized stresses at the roots of the threads.

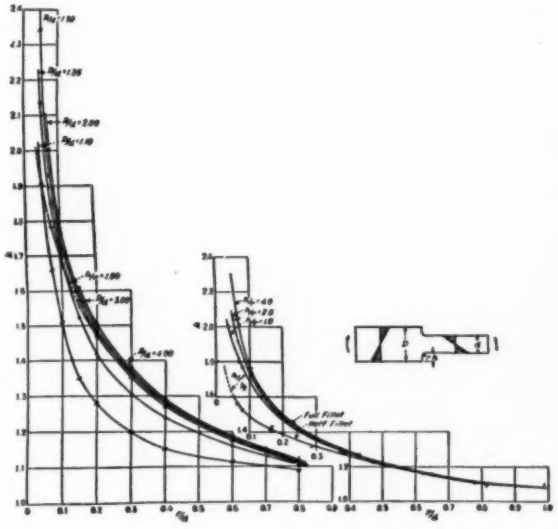


FIG. 14. Values of the stress concentration factor as a function of the radius of curvature of a fillet. These curves are taken from a paper by Frocht in *Mechanical Engineering*.

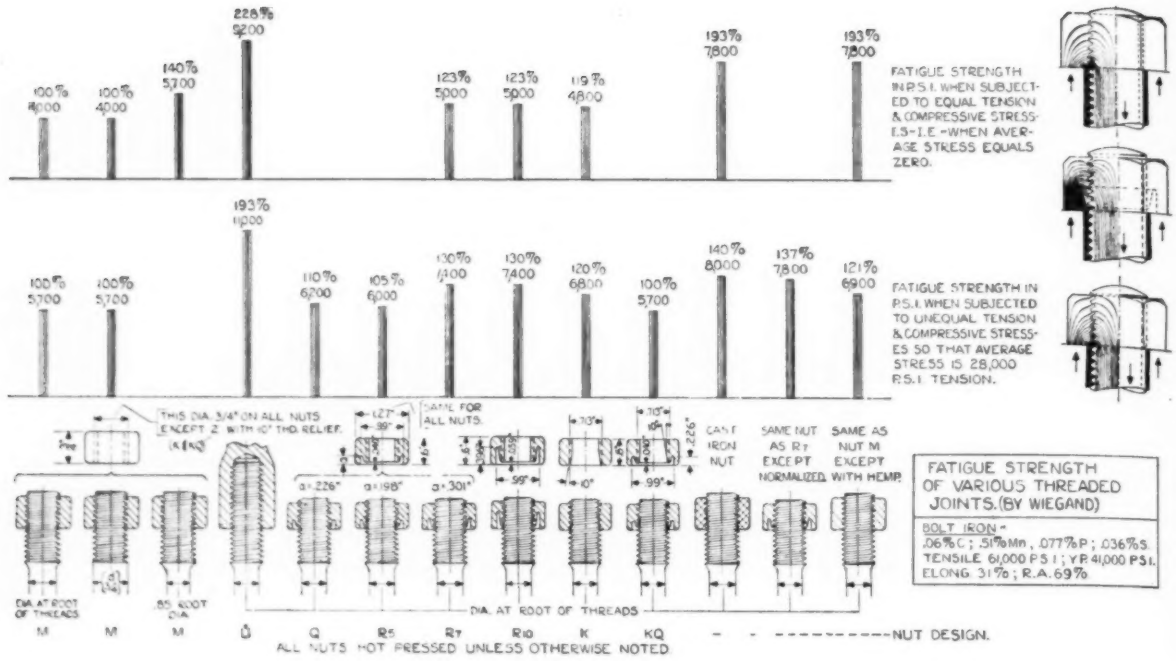

100% 4000
100% 4000
140% 5700
228% 9200
123% 5000
123% 5000
119% 4800
193% 7800
193% 7800
FATIGUE STRENGTH IN P.S.I. WHEN SUBJECTED TO EQUAL TENSION & COMPRESSIVE STRESSES - I.E. - WHEN AVERAGE STRESS EQUALS ZERO.
100% 5700
100% 5700
193% 11000
110% 6700
105% 6000
130% 7400
130% 7400
120% 6800
100% 5700
140% 8000
137% 7800
121% 6900
FATIGUE STRENGTH IN P.S.I. WHEN SUBJECTED TO UNEQUAL TENSION & COMPRESSIVE STRESSES SO THAT AVERAGE STRESS IS 28,000 P.S.I. TENSION.
THIS DIA. 3/4" ON ALL NUTS EXCEPT Z WITH 10° THD. RELIEF.
SAME FOR ALL NUTS.
CAST IRON NUT
SAME NUT AS R7 EXCEPT NORMALIZED
SAME AS NUT M EXCEPT WITH HEMP
FATIGUE STRENGTH OF VARIOUS THREADED JOINTS. (BY WIEGAND)
BOLT IRON - .06% C ; .51% Mn , .077% P ; .036% S. TENSILE 61,000 P.S.I. ; Y.P. 41,000 P.S.I. ELONG. 31% ; R.A. 69%
DIA. AT ROOT OF THREADS
.85 ROOT DIA.
M M M Ü Q R5 R7 R10 K KQ
NUT DESIGN.
ALL NUTS HOT PRESSED UNLESS OTHERWISE NOTED

FIG. 15. Fatigue strength of various threaded joints (by Wiegand).

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# Here and There

### New Appointments and Promotions

Dr. Theodore Jorgenson, Jr., now assistant professor of physics at Clark University, Worcester, Massachusetts, has been appointed assistant professor of physics at the University of Nebraska from September 1, 1938.

★

Dr. A. B. Focke has been promoted from instructor to assistant professor at Brown University.

★

Dr. G. P. Harnwell, associate professor of physics at Princeton, has resigned to take the position of chairman of the department of physics at the University of Pennsylvania. Dr. L. N. Ridenour has also resigned to accept an assistant professorship at the same university.

★

Dr. D. R. Inglis has been appointed assistant professor of physics at Johns Hopkins University. For the past year Dr. Inglis has been at Princeton University on leave of absence from the University of Pittsburgh.

★

The following promotions and new appointments have been made at Princeton: Dr. A. G. Shenstone has been promoted to the Class of 1909 Professorship of Physics; Dr. E. P. Wigner, of the University of Wisconsin, has been appointed to the Jones Professorship of Mathematical Physics; Dr. H. P. Robertson has been promoted to a professorship of mathematical physics; Dr. Walker Bleakney has been promoted to an associate professorship; Dr. M. G. White has been promoted to an assistant professorship; Dr. J. A. Wheeler, of the University of North Carolina, has been appointed to an assistant professorship; Dr. W. F. Brown, of Columbia University, has been appointed to an assistant professorship; and Dr. Joseph Giarratana has been appointed instructor.

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Dr. James Franck of the Johns Hopkins University has been appointed professor of physical chemistry at the University of Chicago. Dr. Franck's appointment was made possible through a grant of $20,000 a year for a period of ten years from the Samuel S. Fels Fund, founded by Mr. Samuel S. Fels, of Philadelphia. The fund provided for the salaries of Dr. Franck, two assistants, technicians and $12,000 for special equipment. Dr. Franck, who was appointed professor of physics at Göttingen in 1920, remained there until the day Hitler assumed power, when he resigned. After six months without a position he moved with his family to Copenhagen, where he was given an appointment at the university. He remained there for a year, after which he accepted his present position of professor of physics at the Johns Hopkins University.

*Science, May 20, 1938*

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Dr. Eugene Paul Wigner, who resigned a year ago from the faculty of Princeton University to become professor of theoretical physics at the University of Wisconsin, has been recalled to Princeton as Thomas D. Jones Professor of Mathematical Physics.

### Charles A. Coffin Fellowships

Nine college graduates have been granted Charles A. Coffin fellowships, totaling $5,000, for the coming school year, it was announced by the General Electric Company. The fellowships are awarded each year to graduates of colleges in the United States who have shown by the character of their work that they could with advantage undertake or continue research work in educational institutions either in this country or abroad. Since 1923 more than 100 students have thus been enabled to make important studies, including one Nobel Prize winner. Charles A. Coffin Fellowships were established by the General Electric Company in 1922 to honor its retiring president and founder.

Of the nine fellowships, three are renewals. J. R. Dietrich, a graduate of William and Mary College and the University of Virginia, is studying for his Ph.D. in the latter institution and will devote himself to a study of the propagation of potential in discharge tubes. R. W. Mattoon, a graduate of Antioch College and the University of Chicago, will continue his study for a Ph.D. degree at Chicago where he will devote himself to measurement of surface contact potentials of built-up stearate films. R. J. Maurer, a graduate of the University of Rochester and a student for his doctor's degree in that school, will study the photoelectric properties of pure sodium.

The six other winners are: S. Bernstein, a graduate of the University of Illinois and of the University of Chicago, where he is studying the application of built-up films to the determination of absolute x-ray wave-lengths, Avogadro's number and the charge of the electron; R. P. Krebs, who graduated from the University of Cincinnati in June, will work on an audio frequency harmonic generator operating on a new principle at his alma mater; M. H. Nichols, a graduate of Oberlin College, is studying for his Ph.D. at M. I. T. where he will continue experimental determination of the "cross section" for excitation of mercury atoms by slow electrons; S. Roberts, studying for his B.S. degree at M. I. T., will investigate the properties of vacuum tube amplifiers having combined positive and negative feedback; W. Waring, a graduate of Cornell University and studying for his Ph.D. at M. I. T., will continue his researches in low temperature thermometry; E. M. Williams, a graduate of Yale University, will continue work there for his Ph.D. on the properties of short-wave transmitting loop antennas.

★

### A. S. T. M. Annual Meeting Program

In the seventeen formal sessions of the Forty-first Annual Meeting of the American Society for Testing Materials held at Chalfonte-Haddon Hall, Atlantic City, June 27 to July 1, inclusive, there were presented 104 technical papers and reports covering important topics on a large number of engineering materials. Dr. A. E. White, A. S. T. M. President, in his annual address discussed "Industrial Research." For many years Director of the Department of Engineering Research, University of Michigan, he considered various questions affecting research in company laboratories and also in the university and related fields.

Dr. Albert Sauveur, Metallurgical Engineer and Professor Emeritus, Harvard University, delivered the Thirteenth Edgar Marburg Lecture. His topic "The Torsion Test" is one on which there is a conspicuous lack of information in the technical literature. He described a specially built torsion machine and discussed results obtained at Harvard. Data covered torsional testing of various steels at room and elevated temperatures in the blue heat range, reverse twisting and the twisting of single crystals. Doctor Sauveur outlined the significance of the autographic torque-twist curve and the desirability of a close study of the torsion test from the standpoint of a routine testing method.

★

### Eastern Photoelasticity Conference

The seventh semi-annual meeting of the Eastern Photoelasticity Conference was held at Harvard University, Cambridge, Massachusetts, on Saturday, May 14, 1938. The fields covered in these photoelasticity conferences are: teaching methods, texts, new materials and polarizers, analytical methods, research projects, plasticity, three-dimensional photoelasticity, and indeterminate structures. Membership in the Conference is entirely informal, and contributions to the meeting are solicited. For further information write to R. W. Vose, Rotch Building, Harvard University, Cambridge, Massachusetts.

★

### Marconi Memorial Award

David Sarnoff, president of the Radio Corporation of America, will be the recipient of the Marconi Memorial Award for 1939. The presentation will be made on April 25 of next year on the anniversary of Marconi's birth. Premier Mussolini has proclaimed the day a national holiday in Italy, and is personally contributing the medal bearing a likeness of Marconi.

★

### Honorary Degrees

Among the honorary degrees to be conferred by the Ohio State University at the commencement exercises on July 4 is the doctorate of science on Dr. Robert B. Sosman, physical chemist of the Research Laboratories of the U. S. Steel Corporation at Kearny, New Jersey, and on Professor Evan J. Crane, editor-in-chief of *Chemical Abstracts*.

*Science*

# Innovations in Instruments

### Sine-Wave Motor-Generator

The General Electric Company has announced a new sine-wave harmonic motor-generator set to fill the need existing in school and college laboratories for a source of sine-wave alternating current of fundamental and harmonic frequencies for testing purposes. Some of the applications for which this set can be used are: meter calibration, iron testing, high-voltage measurements, wave analysis, phase-displacement problems, and the telephone-interference studies. The second-harmonic generator has been included for reproducing certain unsymmetrical wave shapes.

The harmonic motor-generator set consists of: (a) a 10 hp, 230- or 115-volt d-c, shunt wound, 40 deg C continuous, ball bearing, 3600 rpm motor capable of 10% speed variation above and below normal speed; (b) a fundamental or 60-cycle generator rated 5 kva, .95 p-f, 220 volts, 3 phase, 60 cycles, 3600 rpm, 50 deg C continuous, ball bearing; (c) a second-harmonic generator rated 2 kva, .95 p-f, 88 volts, 3 phase, 120 cycles, 3600 rpm, 50 deg C continuous, ball bearing; (d) a third-harmonic generator rated 2 kva, .95 p-f, 88 volts, 3 phase, 180 cycles, 3600 rpm, 50 deg C continuous, ball bearing; (e) a fifth-harmonic generator rated 1 kva, .95 p-f, 44 volts, 3 phase, 300 cycles, 3600 rpm, 50 deg C continuous, ball bearing; and (f) a seventh-harmonic generator rated 1 kva, .95 p-f, 44 volts, 3 phase, 420 cycles, 3600 rpm, 50 deg C continuous, ball bearing.

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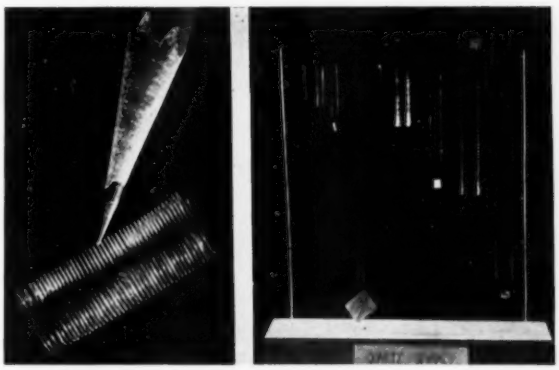
### Glass-like, but Springy

Because it excels the best spring-steel in several respects glass-like clear fused quartz is being used as springs to indicate minute differences in weight in the General Electric research laboratory at Schenectady. Springs made of hair-like filaments of quartz can be stretched to ten times or more their original length and will return almost exactly while steel springs would undergo a permanent stretching. Quartz has an extremely high melting point, and quartz coils can be used at elevated temperatures; steel springs lose their temper at a relatively low temperature. Quartz spirals are not affected by any degree of humidity; steel is subject to corrosion. Quartz is practically invulnerable to the multitude of chemicals encountered in a laboratory; many affect steel. And, finally, quartz coils weigh far less than, and have resulting advantages over, corresponding steel springs.

In the laboratory investigations the quartz springs are usually suspended within a glass tube which is maintained at the desired temperature by immersion in an oil bath. A small glass "boat" suspended from the coil holds the sample under investigation. As the sample varies in weight, so does the length of the quartz spring; and since the length of the spring is proportional to the weight, readings of length give accurate weight values. Measurements are made within an accuracy of a milligram.

The quartz springs are used in measuring weight changes of various materials, under different conditions of heat and humidity. In measuring the moisture absorption of cotton, the sample is suspended from the spring in a vacuum, and the stretch of the coil noted. As water vapor is introduced at different pressures, the amount of absorption can be determined by the increase in length of the spring. Another application has been in measuring the rates of decomposition, in high vacuums, or such materials as organic resins.



The quartz coils used in making the measurements are produced in the G-E research laboratory, out of rods of the fused material as produced in the Thomson Research Laboratory of the company at Lynn, Mass. In the glass-blowing department of the laboratory at Schenectady a small section of a quarter-inch rod of the quartz is heated to more than 3000 deg F with an oxyhydrogen flame. A sudden, straight pull is then applied, whereupon the quartz pulls out into a fiber of about six one-thousandths of an inch diameter—much as molasses candy stretches into threads. These threads, 15 or more feet in length, are calipered, and those within a quarter mil of the desired diameters are saved.

The thread is made into springs by placing it in a long, brass trough leading to a mandrel of the desired diameter. Three small flames of hydrogen in air, giving a temperature of 1800 deg F, soften the thread so that it can be coiled on the mandrel, which is being rotated at two rpm.

Some of the coils measure $\frac{3}{8}$ inch in diameter and $1\frac{1}{2}$ inches in length and have fifty turns to the inch; others are of $\frac{5}{8}$ or $\frac{3}{4}$ inch diameter and have as many as eighty turns to the inch.

★

### G-E Light-Beam Electrostatic Voltmeter

Modern testing and measurement technique has found that older designs of electrostatic voltmeters do not meet present-day requirements. These older types—which are cumbersome, slow in response, requiring calibration in place and affected by stray electrostatic fields—find little use in modern testing, where speed, reliability, ease of use, and portability are demanded.

A new G-E light-beam electrostatic voltmeter is announced. This instrument uses a light-weight, low-inertia moving element. A mirror mounted on its shaft reflects light from a suitable optical system onto a scale, thus affording a high-speed system. The time of response of this voltmeter is four seconds, as compared to about three minutes for the old type. The overall dimensions are $11\frac{7}{8}$ by $12\frac{3}{4}$ by $20\frac{7}{8}$ inches, and the total weight 29 pounds. The case, which is of aluminum, acts also as a shield. It is available in full-scale ratings of from 3 to 20 kv.

★

### Vitamin Therapy Aids Eye Fatigue

The possibility of relieving eye fatigue among certain types of industrial workers and at the same time improving their general health and capacity for work has been the subject of extensive research by Dr. Ralph C. Wise, eye specialist, Mansfield, Ohio, who has just completed some extensive experimental work in cooperation with Dr. O. H. Schettler of the medical department of the Westinghouse Electric and Manufacturing Company. In an article in the "Ohio State Medical Journal" describing the results of the first exhaustive application of Vitamin A therapy to industry, Drs. Wise and Schettler told how three capsules of carotene-in-oil daily, by speeding up the regeneration of "visual purple" and thus increasing the "light threshold" had improved the efficiency of color-matching inspectors in the Westinghouse Merchandising Division more than 75 per cent.

"Visual purple" is a substance in the retina of the eye that is closely connected with the process of seeing. When light falls on the retina, which corresponds to the sensitive plate or film in a camera, the visual purple is gradually changed into another substance, and this change seems to be responsible for the eye's sensitivity. In order to maintain a supply of visual purple the body must continually produce or regenerate it, and this process is known to require the presence of Vitamin A.

The specific problem that led to his study was a relatively high percentage of rejections of off-color parts in the assembly of electric ranges. The present experiments were made with the aid of a new machine known as the bio-photometer, which very accurately measures the rate and regeneration of visual purple by means of obtaining "light thresholds" under standard conditions. When preliminary tests with the bio-photometer showed conclusive evidence of eye fatigue in the color matchers, the physicians proceeded on the assumption that Vitamin A deficiency was clearly indicated and began giving carotene-in-oil, a concentrated source of the vitamin in capsules to correct the condition.

After chartings had been compiled, and frequent re-examinations made with the bio-photometer to determine if the saturation point had been reached, a standard dosage of three capsules of 10,000 units each daily was determined upon, and is now being given.

At the end of seven months they were able to report that the treatment actually did improve the regeneration of visual purple from 50 percent to 100 percent, and that in consequence the new system was saving the company several thousand dollars a year and saving the employes fatigue and weakened vision.

★

### Phototube Relay Unit

A new phototube relay unit suitable for experimental or research work has been developed by the G-M Laboratories, Inc., Chicago. This phototube relay, known as Type 1224A, comprises a phototube, thermionic amplifier, resistor, potentiometer, condenser, and a small electromagnetic relay. The wiring diagram is printed on the base. All wire, terminals, brackets, and screws are included. It is furnished as a kit for student assembly, or completely assembled and ready for use.

Unit 1224A is usable as a very sensitive electronic relay. Not over 1 or 2 microamperes need be controlled in the input circuit to control a 100 watt load by the relay. This makes the device useful for controlling constant temperature baths with a mercury thermostat, since the control circuit places such a small load on the thermostat contact.

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### New Portable D-C Instrument

The General Electric Company has developed a new line of direct current instruments which is particularly adapted to laboratory use. They are known as Type DP-9, and they utilize the new "Concentric-Magnet" principle of operation which is made possible by the development of "Alnico"—a new magnetic material. Alnico makes possible an extremely large air-gap across which is maintained a magnetic field of high intensity. This results in a high-torque instrument with little likelihood of mechanical trouble, even after years of service. Two magnets of Alnico are surrounded by a soft steel ring, and the entire assembly die-cast together. The steel ring serves the dual purpose of completing the magnetic circuit and effectively shielding the instrument against stray magnetic fields.

Voltmeters, ammeters, microammeters, milliammeters, and millivoltmeters are available. This new construction has made possible instruments of exceptionally high sensitivity. Proper damping is obtained by eddy currents set up in the aluminum shell on which the armature coil is wound. An overall accuracy of $\frac{1}{2}$ of one percent of full-scale value is standard for the entire line. Each instrument is equipped with a knife-edge pointed and uniformly divided mirror scale, to eliminate parallax errors and facilitate accurate readings.

# Some of the New Books

**Principles of Radio.** KEITH HENNEY. Third edition. Pp. 495+xii, Figs. 311, 13½×20 cm. John Wiley & Sons, New York, 1938. Price $3.50.

This well-known textbook by the editor of *Electronics* is full of practical information for the student or the intelligent amateur who wishes to increase his comprehension of radio. The book includes discussion of many new tubes and circuits and there are numerous problems and experiments of a simple and suggestive nature. In many instances, the text is clarified by good illustrative examples. The author wisely keeps his readers mindful of the fact that radio deals with actual tubes, coils, and condensers. It is this practical aspect of the book which is its best feature. It meets the needs of the novice by introducing him to the fundamentals of electricity and of alternating current, and by leading him into an understanding of radio practice. However, it seems to the reviewer that the author frequently branches out into complex or incidental matters of modern receiver design before he has laid a sufficiently firm foundation in fundamental circuits.

After the introductory chapters on electricity and alternating current and resonant circuits, the major portion of the book is devoted to the problems of radio receivers. The final three chapters deal with transmitters, modulators, antennae, and television. The author uses, as he has a perfect right to do, the characteristic language of radio (and its jargon), but he frequently forgets to define the terms he uses. The radio amateur might encounter no difficulty on this score, but the novice does.

The reviewer, as a physicist, has a strong negative bias against the introduction of formulae for which there is no adequate explanation in the text. The book contains many equations that have been dragged in by the heels. The student must either ignore them, learn them by rote, or search far afield to find justification for them. In the latter half of the book the student is likely to become mired in a mass of material inadequately explained and poorly organized. The reviewer's experience with his own class (mostly Freshmen) gives him reason to know how baffled moderately intelligent young men do become in studying some of the later chapters. It would seem that the level of exposition is about 10 db below the level of complexity.

RICHARD M. SUTTON
*Haverford College*

**Electron Optics in Television.** I. G. MALOFF AND D. W. EPSTEIN. Pp. 299+xi, illustrated, 15½×23½ cm. McGraw-Hill Book Company, New York, 1938. Price $3.50.

The authors have here collected the results of their extensive experience in design of cathode-ray tubes. The methods presented have been developed principally by the authors themselves, and are highly practical solutions of the problems encountered in design of television cathode-ray tubes.

Following a brief survey of the elements of modern cathode-ray television systems the first part of the book—Electron Optics—is introduced by a description of the fundamental properties of the electron, of gases in evacuated tubes, and of electron emission, thermionic, secondary, and photoelectric. The analogy between the motion of a charged particle in a conservative field of force and the path of a light ray is discussed and applied to the motion of electrons in axially symmetric electric and magnetic fields. The electron optics of the bi-potential lenses encountered in television cathode-ray tubes is considered in great detail, with a wealth of equations and empirical data and curves that should be a delight to any engineer or scientist. A similar treatment is accorded to the defects of electron-focusing and practical methods of measuring and compensating for them are developed.

In the second part—Television Cathode-Ray Tubes—the authors present empirical material on the design of electron guns and beam-deflection systems, with appropriate mathematical analysis where this is possible. The properties of luminescent screens are discussed briefly, being limited substantially to the case of willemite screens. The important factors in rating these tubes are explained, and a chapter is devoted to a discussion of auxiliary circuits and equipment.

The book does not claim to be a comprehensive or profound treatment of electron optics. It is, however, more complete than any other treatment in English familiar to the reviewer. The book does not refer adequately to the rather extensive literature on the subject, and perhaps fails to accord proper credit to the work of Busch and other pioneer workers in this field.

It is an intensely practical account of the application of electron optics to a particular type of cathode-ray tube; and it contains sufficient of the fundamentals so that the reader can understand the problems involved in their application to other types.

C. J. CALBICK
*Bell Telephone Laboratories, Inc.*

**Science and Music.** SIR JAMES JEANS. Pp. 258+x, Figs. 64, 14½×22 cm. Macmillan Company, New York, 1937. Price $2.75.

**Combustion, Flames and Explosions of Gases.** BERNARD LEWIS AND GUENTHER VON ELBE. Pp. 415, Figs. 79. Macmillan Company, New York City, 1938. Price $5.50.

# Among our Authors

C. F. Kettering

O. J. Horger

C. Breer

**Dr.** Warren F. Busse received his Ph.D. from the University of Wisconsin in 1927. During the year 1927–1928 he was a Charles A. Coffin Fellow at the University of Wisconsin. He was with the U. S. Forest Products Laboratory from 1923 to 1924; research associate at the University of Wisconsin from 1927 to 1928. In 1928 he accepted a position as physicist with the B. F. Goodrich Company where at present he is manager of the Physical Research Laboratory.

**Mr.** Charles Franklin Kettering received his degrees of E.E. and M.E. from Ohio State University in 1904. He invented starting, lighting and ignition devices for automobiles which later became known as "Delco System." In 1914 he organized the Dayton Metal Products Company. He is now president and general manager of the General Motors Research Corporation. He is a member of the SAE, ASME, AIEE, American Chemical Society, American Philosophical Society, AAAS, American Academy of Sciences, American Academy of Political and Social Science.

**Mr.** Carl Breer received his B.S. from Leland Stanford University. He has been employed by the Allis-Chalmers Company, Moreland Distillate Truck Company, Acme Electric Auto Works, Studebaker Corporation, Willys Corporation, Zeder-Skelton-Breer Engineering Company, and the Maxwell Motor Corporation. Since 1924 he has been with the Chrysler Corporation and is now director of engineering research. He is a member of the SAE, ASME, Franklin Institute, National Research Council, and vice president of the Chrysler Institute of Engineers.

**Dr.** S. A. Schelkunoff received his B.A. and M.A. in mathematics from the State College of Washington in 1923, and his Ph.D. from Columbia University in 1928. He was connected with the engineering department of Western Electric Company from 1923 to 1925; with the Bell Telephone Laboratories from 1925 to 1926; with the department of mathematics of the State College of Washington from 1926 to 1929. Since 1929 he has been with the Bell Telephone Laboratories. Dr. Schelkunoff has been engaged in mathematical research, especially in the field of electromagnetic theory.

**Dr.** Emory Leon Chaffee received his B.S. from Massachusetts Institute of Technology in 1907, his A.M. and Ph.D. from Harvard University in 1908 and 1911, respectively. He is a member of the American Physical Society, International Scientific Radio Union, American Academy of Arts and Sciences, Institute of Radio Engineers, American Association for the Advancement of Science, and the American Optical Society. At present he is Gordon McKay Professor of Physics and Communication Engineering at Harvard University.

**Dr.** O. J. Horger received his B.S. from Carnegie Institute of Technology in 1923. Since 1923 he has been employed by the Timken Roller Bearing Company, entering service as a draftsman and later engaged in design and research work. During this time he spent three years in graduate study and research work at the University of Michigan where he received the degrees of M.S.E. and Sc.D. in 1934 and 1935, respectively. He now has charge of railway engineering and research with the Timken Roller Bearing Company.

S. A. Schelkunoff

W. F. Busse

E. L. Chaffee

# Contributed Original Research

## The Operating Characteristics of Power Tubes

E. L. CHAFFEE
*Cruft Laboratory, Harvard University, Cambridge, Massachusetts*
(Received February 16, 1938)

This paper describes a new method of precalculating the performance of a power tube from its static characteristic curves. The 60-cycle test method is also briefly outlined and the results of the two methods compared and discussed. An appendix discusses the effect of secondary emission from the plate and grid and primary emission from the grid on the performance of the tube.

A THERMIONIC vacuum tube used as amplifier, oscillator, or modulator is a circuit element which converts power from a unidirectional source into alternating current power. It is important from an economic point of view, especially when large tubes are involved, to operate the tube to the best advantage consistent with the particular conditions and limitations of the case. These conditions may vary widely according to the class of service for which the tube is used. A problem of considerable importance is the choice of the steady and alternating potentials to be applied to the power tube to obtain the prescribed conditions of operation. The purpose of this paper is to describe various methods of solving this problem.

In any particular case there are certain more or less definitely prescribed conditions which must be considered in choosing the proper potentials to apply to the tube. Some of these conditions which are commonly imposed in practice are listed below, but of course they assume different relative importance in different cases. For a power amplifier the average driving power available may be limited, or when distortion is important the peak driving power may be more important than the average driving power. Under some circumstances the power gain is an important consideration. On the other hand, the cost of power may be so important as to make the efficiency a primary consideration. When the tube is used as a modulator, the distortion of the radiofrequency envelope is generally the most important consideration, but always efficiency, power output, and driving power are also important factors.

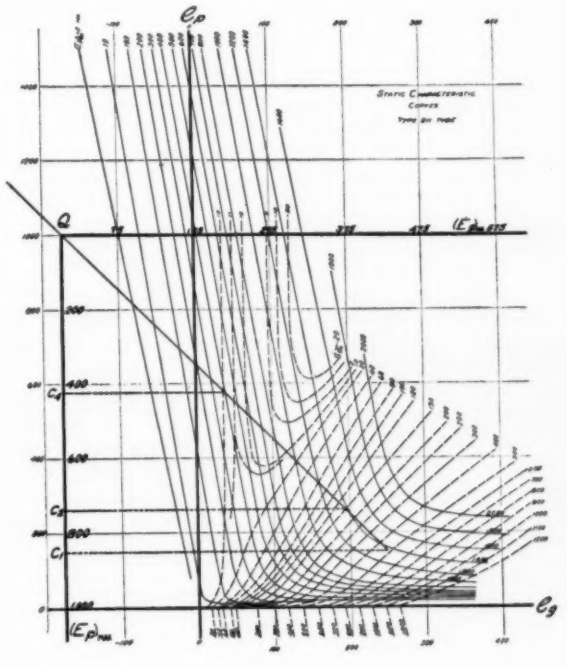

FIG. 1. Static characteristic curves for Slyvania 211, showing a path of operation. ($(I_p)_{Av}$ should read $i_p$. $(I_g)_{Av}$ should be $i_g$.)

Certain limitations are always imposed on the operation of the tube no matter what the class of service or the conditions of the case may be. These limitations are designed to prevent sudden

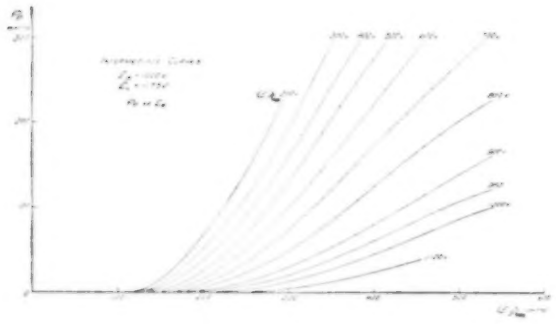

FIG. 2. Intermediate curves of $P_p$.

failure of the tube and to preserve reasonable life. The electrode dissipations are limited to prevent liberation of gas or injury to the tube structure by melting or warping. The grid dissipation must not be so great as to cause appreciable primary emission from the grid.[2, 19] The maximum steady plate potential is limited to prevent breakdown of insulation, flash over due to gas or cold emission, or puncture of the tube caused by stray beams of electrons.

There are, generally speaking, two methods of determining the operating characteristics of a power tube from which can be deduced the proper potentials to be applied to the tube consistent with the limitations and conditions of the case. First, the dynamic operating characteristics of the tube can be calculated from the static characteristic curves of the tube; second, the tube may be tested by some appropriate experimental technique.

There are various methods of precalculating the performance of a power tube. Some of these methods consist of graphical integrations under the current waves obtained from the static characteristic curves.[1, 3, 5, 12, 14, 22] For these methods the static curves must be available far out in the regions of positive grid potentials where the electrode dissipations are greatly in excess of what the tube can withstand for more than an instant. The experimental technique for obtaining these static curves is described elsewhere.[7, 24] In other methods of precalculation the integrations are performed analytically by assuming certain simple expressions for the current pulses or by using approximate analytical expressions for the static characteristic curves.[4, 9, 11, 13, 15, 17, 20, 22]

Some of the methods of precalculation are accurate but extremely laborious while others are rapid but inexact. A new method of calculation* is presented in this paper which is rapid and sufficiently exact for engineering purposes.

The methods of obtaining the operating characteristics of a power tube by direct test fall into two groups. First, the tubes may be operated at a radiofrequency and the powers determined for various d.c. potentials and load resistances. Second, the tube may be tested at a low frequency such as 60 cycles per second where ordinary wattmeters can be used.[15, 18, 23] The high frequency method is less exact and not as flexible as one of the low frequency methods suggested by the author†[15, 23] and described later in this paper.

### A. Calculation of the Operating Characteristics

The method of calculation of the operating characteristics of a power tube now to be de-

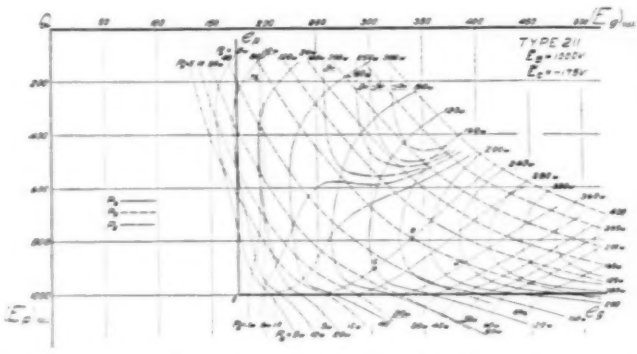

FIG. 3. Contours of $P_b$, $P_{g_1}$ and $P_c$ for a 211 tube. $\bar{E}_B = 1000$ volts, $\bar{E}_C = -175$ volts.

* Described before the Philadelphia section of the I.R.E., June 1936.

† Paper presented before the joint meeting of the I.R.E. and U.R.S.I. in Washington, April, 1934.

scribed depends first upon a complete chart of static characteristic curves similar to that shown in Fig. 1. These curves were obtained by a method described by the author elsewhere.[24]

The $Q$ point (quiescent point) is determined from the values of steady plate and grid potentials ($\bar{E}_B = 1000$ volts and $\bar{E}_C = -175$ volts). The coordinate axes for the instantaneous plate and grid potentials $e_p$ and $e_g$ are the axes for the static curves. Point $Q$ is the origin from which the maximum alternating plate and grid potentials, $(E_p)_{max}$ and $(E_g)_{max}$, are measured.

The alternating plate and grid potentials are assumed to be sinusoidal and 180 degrees in phase relation. Under these conditions the path of operation, as shown in Fig. 1, is a straight line through the $Q$ point with a slope depending upon the relative values of the potential amplitudes $(E_p)_{max}$ and $(E_g)_{max}$.

Now assume a particular path shown in the diagram corresponding to certain values of $(E_p)_{max}$ and $(E_g)_{max}$. We are to calculate for this particular path the power input to the plate circuit, $P_B$, the power output, $P_b$, the apparent plate dissipation, $P_p$, the plate-load resistance, $(R_b)_\omega$, the plate-circuit efficiency, $(Eff)_p$, the grid-driving power, $P_d$, the apparent grid dissipation, $P_g$, and the power amplification, $(P.A.)$.

In order to calculate the quantities listed above it is necessary to obtain the average plate and grid currents, $(I_p)_{Av}$ and $(I_g)_{Av}$, and the amplitudes of the fundamental alternating components $(I_p)^1_{max}$ and $(I_g)^1_{max}$. The plate and grid currents could readily be plotted against time from the chart of Fig. 1 but that is unnecessary. A simple

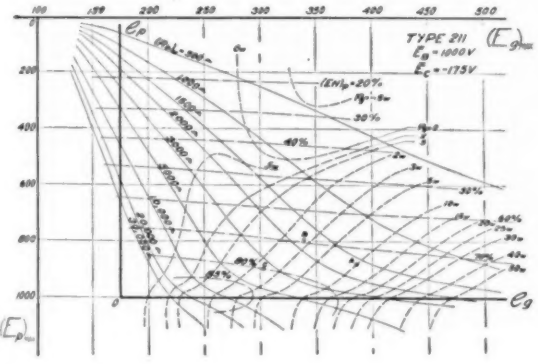


Fig. 4. Contours of $(Eff)_p$, $P_g$, and $(R_b)_\omega$ for a 211 tube. $\bar{E}_B = 1000$ volts, $\bar{E}_C = -175$ volts.

method of harmonic analysis described elsewhere by the author[21] ‡ can be performed directly. Using only that part of the harmonic analysis

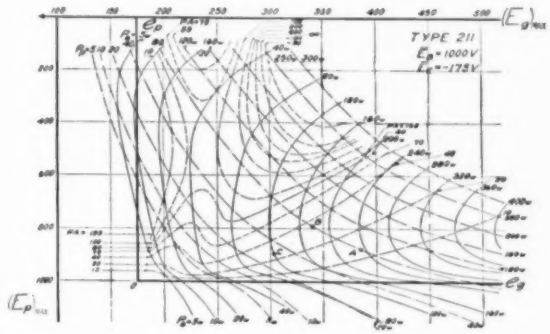


FIG. 5. Contours of $P_b$, $P_p$, and $(P.A.)$ for a 211 tube. $\bar{E}_B = 1000$ volts, $\bar{E}_C = -175$ volts.

which is required for this calculation, we proceed as follows, using the eleven-point analysis. Draw horizontal lines at $(E_p)_{max}$, 0.866 $(E_p)_{max}$, and 0.5 $(E_p)_{max}$ as shown in Fig. 1 by the lines marked $C_1$, $C_2$, and $C_4$. Call the instantaneous plate currents corresponding to the intersection of these horizontal lines and the path of operation $i_p'$, $i_p''$, and $i_p'''$. Indicate the current, if any, at the $Q$ point by $i_p^{iv}$. Then according to the schedule in the paper just referred to,

$$(I_p)_{Av} = (1/12)(i_p' + 2i_p'' + 2i_p''' + 2i_p^{iv}) \quad (1)$$

$$(I_p)^1_{max} = (1/6)(i_p' + (3)^{\frac{1}{2}} i_p'' + i_p'''). \quad (2)$$

These relations are special deductions from the harmonic analysis for this particular use. They are valid only if the current is zero for all points corresponding to $i_p'$, $i_p''$, and $i_p'''$ on the other half-cycle. This is practically always true for class $B$ and class $C$ operation, but not for class $A$ operation. For calculation of class $A$ operation the original paper[21] should be consulted and the complete forms of harmonic analysis used.

Since the grid current is practically always zero at the $Q$ point, the expressions for average

‡ A correction has been published in the July number, 1937, of R.S.I. affecting the value of $C$ listed in the second column of page 386. The correct values are:

$$C_2 = \frac{\sqrt{3}}{2}; \quad C_3 = \frac{1}{\sqrt{2}}; \quad C_4 = \frac{1}{2}; \quad \text{and} \quad C_5 = \frac{(2-\sqrt{3})^{\frac{1}{2}}}{2}.$$

grid current and for the maximum value of the fundamental component of grid current are

$$(I_g)_{Av}=(1/12)(i_g'+2i_g''+2i_g''') \quad (3)$$

$$(I_g)^1{}_{max}=(1/6)(i_g'+(3)^{\frac{1}{2}}i_g''+i_g'''). \quad (4)$$

Assume now that $(I_p)_{Av}$, $(I_p)^1{}_{max}$, $(I_g)_{Av}$, and $(I_g)^1{}_{max}$ for the particular path have been evaluated. The four quantities $P_b$, $P_p$, $P_d$, and $P_g$ are now calculated from the obvious expressions listed below.

$$P_b=\frac{(E_p)_{max}(I_p)^1{}_{max}}{2}=\text{Power output} \quad (5)$$

$$P_p=\bar{E}_B(I_p)_{Av}-P_b=\text{Apparent plate dissipation} \quad (6)$$

$$P_d=\frac{(E_g)_{max}(I_g)^1{}_{max}}{2}=\text{Driving power} \quad (7)$$

$$P_g=\bar{E}_c(I_g)_{Av}+P_d=\text{Apparent grid dissipation } (\bar{E}_c \text{ is a negative quantity}). \quad (8)$$

A sample calculation for the particular path shown in Fig. 1 is given below. This path ends at $(E_p)_{max}=850$ volts and $(E_g)_{max}=425$ volts.

| Plate | Grid |
|---|---|
| $i_p'=1.365$ amp. | $i_g'=0.485$ amp. |
| $i_p''=1.380$ " | $i_g''=0.150$ " |
| $i_p'''=0.400$ " | $i_g'''=0.010$ " |
| $i_p{}^{iv}=0$ " | |

$$(I_p)^1{}_{max}=\frac{1.365+(3)^{\frac{1}{2}}\times 1.380+0.800}{12}=0.692 \text{ amp.}$$

$$(I_p)_{Av}=\frac{1.365+2.760+0.800}{12}=0.410 \text{ amp.}$$

$$P_b=\frac{850\times 0.692}{2}=294 \text{ watts}$$

$$P_p=1000\times 0.410-294=116 \text{ watts}$$

$$(I_g)_{Av}=\frac{0.485+0.300+0.020}{12}=0.0671 \text{ amp.}$$

$$(I_g)^1{}_{max}=\frac{0.485+(3)^{\frac{1}{2}}\times 0.150+0.020}{6}=0.0126 \text{ amp.}$$

$$P_d=\frac{425\times 0.0126}{2}=26.8 \text{ watts}$$

$$P_g=-175\times 0.0671+26.8=15.0 \text{ watts.}$$

The calculations just outlined for the particular path assumed are repeated for a number of other paths all having the same $(E_p)_{max}$ but different values of $(E_g)_{max}$. For these paths the same horizontal lines used to find the values of $i'$, $i''$, and $i'''$ can be used.

The resulting values of $P_b$, $P_p$, $P_d$, and $P_g$ are plotted against $(E_g)_{max}$ for the several constant values of $(E_p)_{max}$ as parameters, giving four sets of curves called intermediate curves. A set of intermediate curves for $P_p$ is shown in Fig. 2.

The intermediate curves are used to obtain the loci of $(E_p)_{max}$ *vs.* $(E_g)_{max}$ for various constant values of $P_b$, $P_p$, $P_d$, and $P_g$. Final contour loci are then plotted on the same axes as those of Fig. 1.

Figure 3 shows a calculated set of contours for $P_b$, $P_p$ and $P_d$ for the tube for which the static curves of Fig. 1 were taken. These contours give the loci of the end of the path of operation for the various constant values of $P_b$ or $P_p$. The steady plate and grid potentials are 1000 and $-175$ volts for this case.

Figure 4 gives the contours of $P_g$ and $(Eff)_g$ for the same conditions as for Fig. 3. The efficiency contours can be obtained by drawing an intermediate diagram as for $P_b$ and $P_p$, but time is saved by drawing the efficiency contours directly from those of $P_b$ and $P_p$, using the relation

$$(Eff)_p=P_b/(P_b+P_p). \quad (9)$$

In Fig. 4 contours of $(R_b)_\omega$ are shown. These can be obtained by means of intermediate

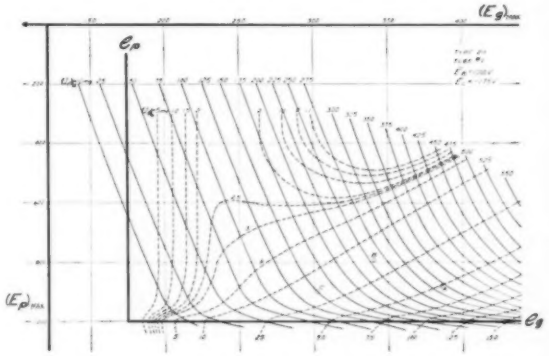

FIG. 6. Contours of $(I_p)_{Av}$, $(I_g)_{Av}$ for a 211 tube. $\bar{E}_B=1000$ volts, $\bar{E}_C=-175$ volts.

graphs but, as for efficiency, these contours of $(R_b)_\omega$ can more conveniently be plotted directly on a piece of tracing paper placed over the

contours of Fig. 3 and then transferred to Fig. 4, using the relation

$$(R_b)_w = (E_p^2)_{\max}/2P_b. \qquad (10)$$

It is a simple matter to locate the $(E_p)_{\max}$ on each $P_b$ contour for a given constant value of $(R_b)_w$.

Figure 5 shows the contours of $P_b$ and $P_p$ of Fig. 3, and in addition, the contours of power amplification ($P.A.$) obtained directly from the contours of $P_b$ and $P_d$, where

$$(P.A.) = P_b/P_d. \qquad (11)$$

Other contours can be derived from the calculations outlined above. For example, a very useful chart is that shown in Fig. 6 in which contours of constant average plate and grid currents are drawn.

Computation can be greatly aided by the use of a mechanical device shown in Fig. 7. A collapsible parallelogram formed of brass strips has fine wires which extend across the parallelogram parallel to two sides at distances the ratios of which are proper for the harmonic analysis used. For the calculations on power tubes described above only three wires are required places at distances proportional to the numbers 0.50, 0.866, and 1. These wires, held taut by means of springs, form the horizontal lines used in the harmonic analysis. A thread or wire is attached at the $Q$ point and can be laid over these horizontal wires to represent any path of operation. By changing the altitude of the parallelogram the lower wire can be set for any value of $(E_p)_{\max}$. The wires are shown attached to separate pieces of metal the positions of which can be set at definite marked points along the side pieces. In order to cover the whole static characteristic curve chart without requiring an awkwardly large parallelogram, it is convenient to be able to set the wires at various distances from the upper bar, always keeping the ratios of distances from the $Q$ point to the wires the same.

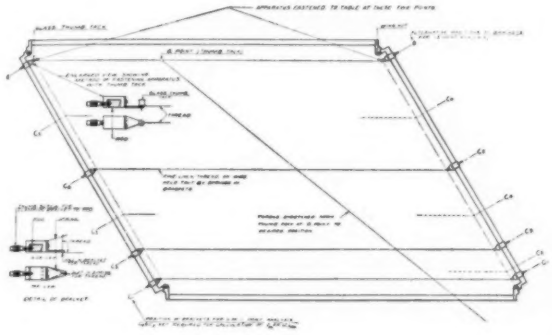

FIG. 7. Mechanical device for aiding in harmonic analysis.

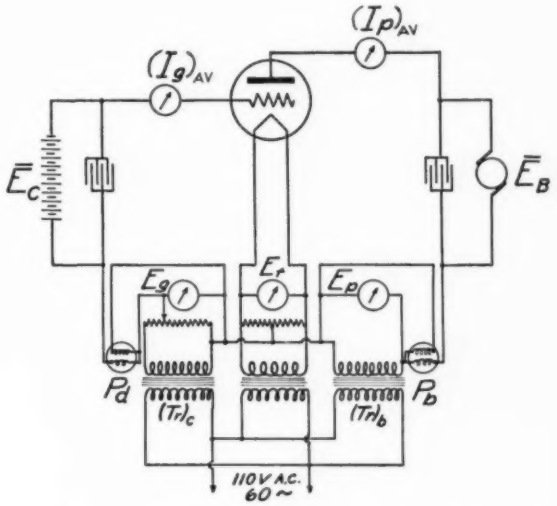


FIG. 8. Simple diagram of connections for testing a power tube at 60 cycles.

## B. Sixty-Cycle Test Method

The performance charts, Figs. 3–6, can be obtained by direct experimental test instead of by calculation. A method of test at a low frequency permitting the use of ordinary measuring instruments has already been described[18] and so will now be only briefly outlined, stressing certain improvements in apparatus and method.

From the point of view of the tube, its performance is determined solely by the potentials impressed upon it, provided the frequency at which it operates is not so high that the transit time of an electron through the tube is an appreciable fraction of the period of oscillation. The tube functions in the normal manner provided suitable steady and sinusoidal alternating potentials are impressed on grid and plate, the alternating potentials being 180 degrees out of phase. One way of obtaining the proper potentials to apply to the tube is that adopted by Spitzer.[2] He built a huge tank circuit, resonant at the 60-cycle

frequency, for the plate load, and applied a sinusoidal grid potential from a transformer.

About ten years ago the author suggested the use of a transformer in the plate circuit to replace the tank circuit. Since the harmonic components of the plate and grid currents set up negligible potentials across the plate and grid transformers because of their low impedance to these currents, the alternating components of the plate and grid potentials are essentially sinusoidal. The use of the transformer has the advantage of requiring no tuning, of being less expensive, and more flexible in operation.

The method using transformers will now be described. Referring to Fig. 8, the grid and plate transformers $(Tr)_c$ and $(Tr)_b$ are operated on the 60-cycle mains. The two d.c. sources, $\bar{E}_C$ and $\bar{E}_B$ may be generators or batteries but must be shunted by suitable condensers to by-pass the alternating components of current. Voltmeters, not shown, are provided to give the values of these steady potentials.

Wattmeters interposed as shown in Fig. 8 give the alternating power fed into the grid circuit, which is $P_d$, and power fed from the plate circuit into the transformer and line, which is $P_b$. The tube converts power drawn from $\bar{E}_B$ into alternating current power $P_b$. Direct-current ammeters read the average currents $(I_p)_{Av}$ and $(I_g)_{Av}$, and voltmeters $E_g$ and $E_p$ give the alternating components of grid and plate potentials.

An improved circuit and the one actually used in the tests to be described is a push-pull arrangement as shown in Fig. 9. This arrangement has the advantages that the fundamental and all odd harmonic components of current are balanced out of the d.c. sources so that only the even harmonics need be by-passed, and also that the steady component of current does not magnetize the cores of the transformers. The d.c. potential for the grid bias is supplied from a self-regulated rectifier system which gives a very stable and continuously variable potential. Batteries are used for potentials below those obtainable from the regulated source or a generator for very high potentials. The alternating potentials are continuously adjustable by means of variacs or adjustable auto transformers in the primary circuits.

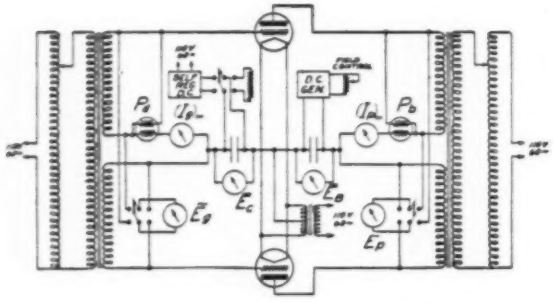

FIG. 9. Complete diagram of connections of the push-pull arrangement of testing a power tube at 60 cycles.

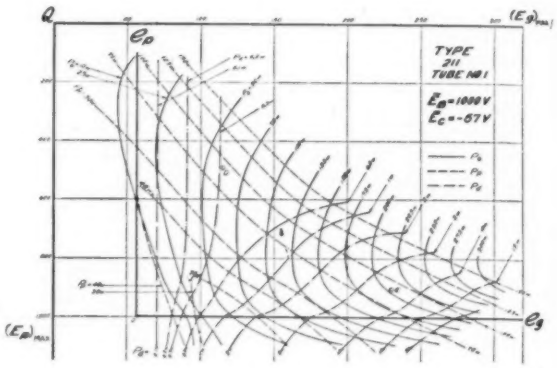


FIG. 10. Contours of $P_b$, $P_p$, and $P_d$ for a 211 tube, used as a linear amplifier. $\bar{E}_B = 1000$ volts, $\bar{E}_C = -57$ volts.

The wattmeters give $P_b$ and $P_d$ directly. The apparent plate and grid dissipations are then given by Eqs. (6) and (8). The plate-circuit efficiency and $(R_b)_\omega$ are calculated as before by Eqs. (9) and (10). The data for $P_b$, $P_d$, $P_p$, and $P_g$ are plotted in intermediate curves from which contours for constant values of the ordinates are derived in exactly the same manner as described under section A. The contours for $(Eff)_p$ and $(R_b)_\omega$ are derived directly rather than by plotting intermediate curves.

The accuracy of the methods of test and calculation is indicated by the check between observed and calculated values for Figs. 3–6. These charts were derived by calculation from the static characteristic curves as described. When the tube was tested by the system of Fig. 9 the experimentally observed points checked the charts for $P_b$, $P_d$, $P_p$, and $P_g$ as closely as the instruments could be read, or well within 1 percent, over as much of the charts as could be

covered without exceeding a plate loss of 150 watts or a grid loss of 20 watts, except for the small region of negative plate potentials and for grid potentials less than 100 volts, where the check was within 3 percent. This unimportant region is one in which the data for calculation from the static curves are scant. This close check demonstrates the consistency of the two methods of obtaining the operating charts and shows that it is immaterial so far as accuracy is concerned which method is adopted for obtaining the operating characteristics. The experimental method is considerably more rapid, but for very large tubes the cost of the equipment for the 60-cycle method is considerably greater than that for taking the static characteristic curves. Hence the method of calculation is probably more practicable for large tubes such as the water-cooled types.

### C. Discussion of the Charts

Both the method of calculation and the 60-cycle test method described above can be used to determine the operating characteristics of a tube used in any way. Oscillator characteristics are easily obtained if contours of $P_0$, equal to $P_b - P_d$, are plotted in place of $P_b$ as shown in the paper by the author and Dr. Kimball.[19] The amplitude of oscillation and the stability of oscillation can be derived. The modulation characteristics of a tube can likewise be derived by using $\bar{E}_B$ or $\bar{E}_C$ as a variable. It is outside the scope of this paper to enter into a discussion of the many types of charts which can be obtained and to discuss at length the deductions from the charts.

A few remarks may be of interest regarding some of the charts shown in order that the reader may understand the application of this contour method of presentation of the tube characteristics. Refer first to Fig. 3. It is obvious that this chart extends out to values of $P_p$ far greater than the tube could withstand even for a short time. The curves for large values of $P_p$ and $P_b$ were drawn in merely to show the trend of the curves. Let us suppose merely for illustration that 80 watts is the limit of safe plate dissipation. If we follow along the $P_p$ line for 80 watts we obtain more power output as we proceed to the right until a maximum of about 270 watts is reached. But Fig. 4 shows that grid dissipation increases rapidly as we proceed to the right. Grid dissipation is another factor which may limit the point of operation. The line marked *gg* in Fig. 3 is the locus of points at which the primary grid emission

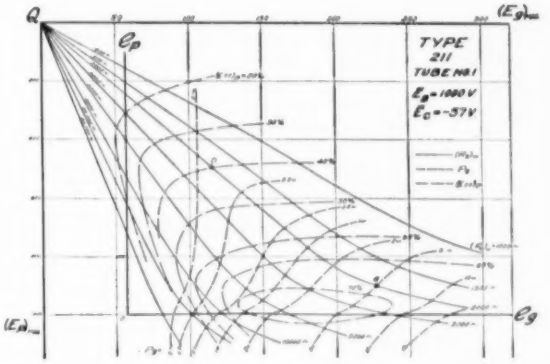


Fig. 11. Contours of $P_{g_1}$ $(R_b)_\omega$ and $Eff)_p$ for a 211 tube used as linear amplifier. $\bar{E}_B = 1000$ volts, $\bar{E}_C = -57$ volts.

is 100 microamperes. Points to the right of this line correspond to greater grid emission. The choice of 100 microamperes as a measure of primary grid emission is quite arbitrary, but at least the line shows the locus over which the temperature of the grid is approximately constant and sufficient to cause this emission.

If the grid dissipation be limited to 10 watts, we can operate at the point marked *A* in Figs. 3, and 4. This point gives a power output of 235 watts at a plate-circuit efficiency of 74.5 percent. From Fig. 3 we obtain the average driving power as 19 watts, and from Fig. 4 $(R_b)_\omega$ as 1650 ohms. The power amplification is 235/19 or 12.3 as shown by Fig. 5. The maximum values of the alternating potentials which correspond to this point of operation are $(E_g)_{\max}$ equals 386 volts and $(E_p)_{\max}$ equals 881 volts. The maximum instantaneous grid potential is $386-175$ or 211 volts, and the minimum plate potential is $1000-881$ or 119 volts. Examining the static curves we see that during operation the plate current rises to a maximum of about 1.03 amperes and has a depression down to about 0.88 ampere in the center of the plate current pulse. The peak grid current is 0.55 ampere. The total emission of this tube may be, when fully activated, over 2 amperes, and is generally well over 1.5 amperes so that operation at the point specified seems to

be fairly conservative in respect to the peak current required. The choice of this point is used merely as an illustration of the use of the chart and is not a recommendation for the operation of this tube.

The average plate and grid currents are generally more convenient quantities than $(E_p)_{max}$ and $(E_g)_{max}$ to use to determine whether or not the tube is operating at any prescribed point. We can easily calculate the average plate and grid currents corresponding to operation at point $A$. The power input is $P_b+P_p$, or 315 watts, which, divided by 1000, gives $(I_p)_{Av}$ equal to 0.315 ampere. Similarly, $-(I_g)_{Av}\bar{E}_C$ is equal to $P_d-P_g$, or 9 watts, and since $\bar{E}_C$ is $-175$ volts, $(I_g)_{Av}$ is 52 milliamperes. These average currents can be obtained directly from Fig. 6.

Point $A$ would seem a satisfactory point of operation except for the fact that to drive the grid as far as 211 volts positive requires the very large peak instantaneous driving power of 386 $\times 0.55$ or 212 watts, whereas the average driving power is only 19 watts. Obviously the driving source would have to have a large reserve capacity in order to maintain sinusoidal wave form with such a large variation of instantaneous power.

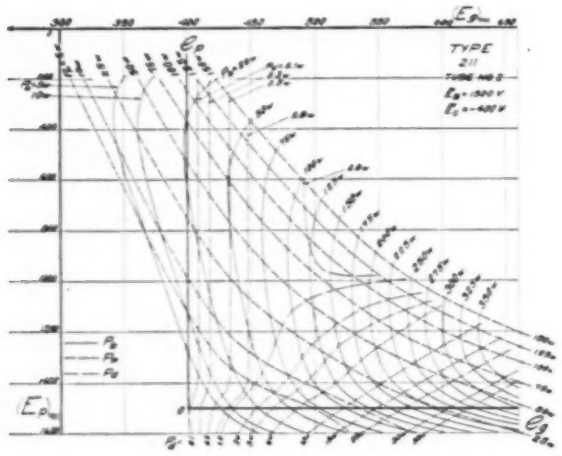

FIG. 12. Contours of $P_b$, $P_p$, and $P_g$ for a 211 tube. $\bar{E}_B=1500$ volts, $\bar{E}_C=-400$ volts.

To reduce the driving power, still retaining a maximum limit of plate dissipation of 80 watts, we proceed to the left from point $A$ along the $P_p$ line for 80 watts until we reduce the average driving power to a reasonable value, say to 5 watts. For this point of operation, denoted as point $B$, the power output is 177 watts. The plate

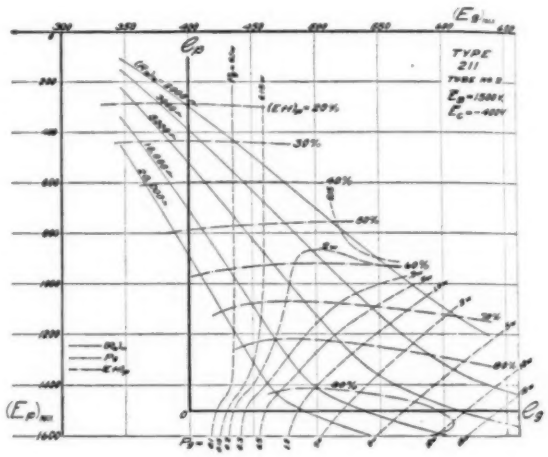

FIG. 13. Contours of $P_g$, $(R_b)_\omega$ and $(Eff)_p$ for a 211 tube. $\bar{E}_B=1500$ volts, $\bar{E}_C=-400$ volts.

efficiency is 69 percent and the plate-load resistance $(R_b)_\omega$ is 1770 ohms. The value of $P_g$ is about 2.7 watts. The peak grid current is 135 milliamperes, giving a peak instantaneous driving power of 46 watts, as compared with 212 watts for point $A$. Because of the lower peak driving power required, point $B$ is a more practical point of operation than point $A$.

If one were satisfied with less power output, and if $P_d$ were still limited to 5 watts, the tube could be made to operate at considerably higher efficiency by choosing, for example, point $C$. For this point $P_b$ is 120 watts and the efficiency is 80 percent. The apparent plate dissipation is only 30 watts and the plate-load resistance 3360 ohms. Although the average driving power is 5 watts, the peak driving power is 50 watts.

When considering the several points of operation it was assumed that the amplitude of the alternating grid potential is constant as in telegraphy or in the amplification of the carrier current before modulation. If the tube is to be used as an amplifier of a wave of varying amplitude such as in carrier or radio telephony, the operating point is constrained to move along the $(R_b)_\omega$ line corresponding to the resistance of the plate load. For linear amplification (class $B$) this

line should be straight and pass through the $Q$ point. Obviously none of the lines of Fig. 4 satisfies this condition because the grid bias is too great.

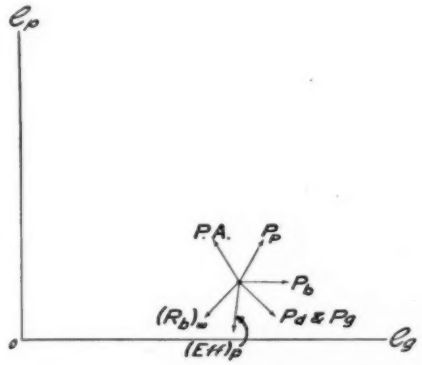


FIG. 14. Direction in which various quantities increase most rapidly. $\bar{E}_B = 1000$ volts, $\bar{E}_C = -175$ volts.

Figures 10 and 11 give contours for a bias of $-57$ volts, the value found by experiment which makes the resistance line for an $(R_b)_\omega$ of 2000 ohms most nearly straight. These contour diagrams were obtained by the 60-cycle test method.

Referring now to Fig. 11, and assuming a plate-load resistance of 2000 ohms, the maximum excursion of the operating point for linear amplification may be taken somewhat around the bend of the 2000-ohm line, thus permitting a slight amount of distortion at high degrees of modulation in order to obtain a greater output from the tube than would be the case if the excursion of the operating point were confined entirely to the straight portion of the resistance line. Assume, then, that the end point of the excursion for 100 percent modulation is at an $(E_g)_{max}$ of 230 volts and an $(E_p)_{max}$ of 905 volts. This point is indicated in Fig. 11 by the letter $e$. The carrier point would thus be at $(E_g)_{max}$ equal to 115 volts and at $(E_p)_{max}$ equal to 490 volts, indicated by point $D$ in Figs. 10 and 11. The carrier conditions are then $P_p$ equal to 92 watts, $P_b$ equal to 60 watts, and the efficiency is 39.5 percent. The average driving power for the carrier with no modulation is slightly over 0.3 watt, whereas the average driving for the peak of the modulation cycle is that for point $e$ or 7.5 watts.

The harmonic analysis[21] can now be applied to determine the average value of the various quantities over the modulation cycle when the degree of modulation is 100 percent. Using the 11-point analysis, the average $P_p$ is 81 watts, average $P_b$ is 85 watts, average $(Eff)_p$ is 51.1 percent, and average $P_d$ is 2.1 watts.

The harmonic analysis applied to the path of operation covered during the modulation cycle, that is, the resistance line for 2000 ohms as limited by the grid swing of 230 volts, gives the amplitude of the fundamental component of plate potential as 471 volts. The amplitude of the second harmonic is 21.8 volts; that of the third 14 volts, of the fourth 13.8 volts, the fifth 2.5 volts, and the sixth 2.9 volts. Higher harmonics are very small. The distortion calculated from these values is 6.3 percent.

Higher efficiency and greater power output for class $C$ operation can be obtained by increasing the d.c. plate potential and the grid bias as shown by the contours of Figs. 12 and 13. These diagrams are for $\bar{E}_B = 1500$ volts and $\bar{E}_C = -400$ volts. For example, Fig. 12 shows that for a plate power of 80 watts and a driving power of 5

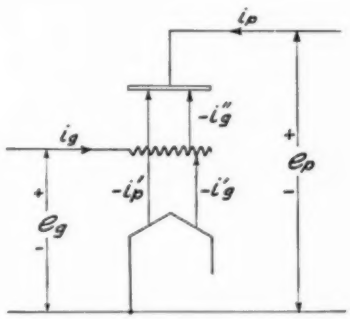


FIG. 15*a*. Secondary emission from grid.

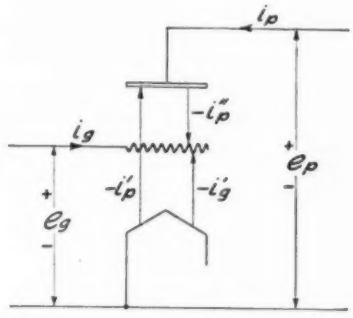


FIG. 15*b*. Secondary emission from plate.

watts, the power output is about 240 watts as compared with 177 watts for operation at point $B$ of Fig. 3 for which the plate and driving powers are the same.

A similarity in shape of the contours for various values of $\bar{E}_B$ and $\bar{E}_C$ is apparent, especially for the region of normal operation. It is perhaps of some value to generalize by showing, as in Fig. 14, the direction in which, for any given value of $\bar{E}_B$ and $\bar{E}_C$, the various quantities increase most rapidly starting from a point in the practical region of operation. Fig. 14 is drawn for an $\bar{E}_B$ of 1000 volts and an $\bar{E}$ of $-175$ volts although the directions are practically the same independent of the values of these voltages.

The use of this diagram is illustrated by the following examples. Suppose that for a certain condition of operation the plate loss is excessive, and it is desired to reduce $P_p$ without reducing the power output $P_b$. The operating point should then move vertically downward, which can be accomplished by increasing the resistance $(R_b)_\omega$, allowing the excitation potential $(E_g)_{\max}$ to remain the same. The efficiency, $P_d$, and $P_g$ will increase. If now $P_d$ is too great and it is desired to reduce it leaving $P_p$ the same, the operating point should move upward and to the left in a direction perpendicular to $P_p$. To effect this change $(R_b)_\omega$ remains about the same but the excitation potential $(E_g)_{\max}$ is reduced. For this change $P_b$ decreases, $(P.A.)$ increases, and the efficiency decreases slightly.

Only a few of the many deductions which can be made from the charts have been given. Many uses of the chart not specifically mentioned here will occur to the reader. These charts furnish complete data for the operation of the tube as an amplifier with fixed bias. A set of charts can be drawn showing the results when the bias is obtained by the flow of the average grid or plate current through a resistance. Other charts can be calculated or obtained directly by experiment at 60 cycles, which give complete engineering data for the tube acting as a self-excited oscillator or as a modulating amplifier. These will be illustrated in later publications.

## Appendix I

### The true electrode dissipation

Attention is called to the word "apparent" in Eqs. (6) and (8) giving $P_p$ and $P_g$. These quantities are the true powers delivered to the plate and grid, but they give dissipations at the electrodes only when there is negligible secondary or primary emission at both plate and grid, and provided there is no appreciable positive ion current. This statement deserves further explanation.

Assume first that secondary or primary emission occurs at the grid as indicated by the electron current $-i_g''$ in Fig. 15a. If $i_g$ represents the total instantaneous positive current to the grid, and $-i_g'$ represents the component of grid current due to the electrons that come from the cathode, then

$$i_g = i_g' - i_g''. \tag{12}$$

Let $e_g$ be the instantaneous grid potential. The actual heat loss at the grid, neglecting the small heating or cooling effect due to the passage of electrons through the surface potential barrier, is

$$\begin{aligned}\text{Grid dissipation} &= \frac{1}{2\pi}\int_0^{2\pi} i_g' e_g d\theta \\ &= \frac{1}{2\pi}\int_0^{2\pi} i_g e_g d\theta + \frac{1}{2\pi}\int_0^{2\pi} i_g'' e_g d\theta \\ &= P_g + \frac{1}{2\pi}\int_0^{2\pi} i_g'' e_g d\theta. \end{aligned} \tag{13}$$

Eq. (13) shows that the actual grid dissipation is greater than the apparent grid dissipation $P_g$. Fig. 4 shows, however, that $P_g$ can be negative in the region of secondary emission from the grid. This is possible only if the integral term of Eq. (13) is greater than the grid dissipation which of course is always positive. When $P_g$ is negative $P_d$ is more negative because the bias source gives out power instead of absorbing it.

Considering now the dissipation at the plate, the actual heating at the plate is

$$\begin{aligned}\text{Plate dissipation} &= \frac{1}{2\pi}\int_0^{2\pi} i_p' e_p d\theta + \frac{1}{2\pi}\int_0^{2\pi} i_g''(e_p - e_g) d\theta \\ &= \frac{1}{2\pi}\int_0^{2\pi} (i_p' + i_g'') e_p d\theta - \frac{1}{2\pi}\int_0^{2\pi} i_g'' e_g d\theta \\ &= P_p - \frac{1}{2\pi}\int_0^{2\pi} i_g'' e_g d\theta. \end{aligned} \tag{14}$$

Hence the plate is less heated than is indicated by the power delivered to the plate.

Assume now that secondary emission occurs at the *plate* as illustrated in Fig. 15b. Then

$$\begin{aligned}\text{Grid dissipation} &= \frac{1}{2\pi}\int_0^{2\pi} i_g' e_g d\theta + \frac{1}{2\pi}\int_0^{2\pi} i_p''(e_g - e_p) d\theta \\ &= P_g - \frac{1}{2\pi}\int_0^{2\pi} i_p'' e_p d\theta. \end{aligned} \tag{15}$$

$$\text{Plate dissipation} = \frac{1}{2\pi}\int_0^{2\pi} i_p' e_p d\theta$$

$$= P_p + \frac{1}{2\pi}\int_0^{2\pi} i_p'' e_p d\theta. \quad (16)$$

Eqs. (15) and (16) show that when secondary emission occurs at the plate, the grid is less heated and the plate more heated than indicated by the power inputs to these electrodes.

The statements just made concerning the effects of secondary or primary emission from the grid or plate compare the actual electrode dissipations with the apparent dissipations or actual power delivered to the electrodes. From another point of view, consider a tube operating at certain potentials with and then without secondary emission. Secondary or primary emission from the grid increases the heating of the plate but does not appreciably affect the heating of the grid. Secondary emission from the plate increases the heating of the grid but does not appreciably affect the heating of the plate. Secondary emission from the plate has an effect only when the maximum grid potential is greater than the minimum plate potential, which occurs only in those regions of Figs. 3, 4, and 5 below a line drawn through the origin for the static curves and whose equation is $e_p = e_g$. Most practical points of operation occur near or above this line so that overheating of the grid due to secondary emission from the plate is generally not serious.

It might be well at this point to summarize the facts concerning secondary and primary emission from the electrodes. Secondary emission from the grid reduces the net grid current and hence reduces the driving power. Although the grid dissipation is greater than indicated by a wattmeter or any method of calculation of grid power, secondary emission from the grid does not increase the grid heating but rather it decreases the apparent grid loss. Secondary emission from the grid may cause blocking and parasitic oscillations if the path of operation passes over much of the negative-grid-current region but this generally occurs only when the plate-load resistance is too low for best operation or when the grid-bias potential is very small. The plate is heated less than indicated by the value of $P_p$ but is actually more heated than would be the case if secondary emission were absent. The calculations from the static curves and measurements by the 60-cycle test method give correct results for $P_p$ and $P_g$ including the effects of secondary emission.

Secondary emission from the plate increases the driving power by increasing the net grid current. Although the grid is less heated than indicated by $P_g$ it is actually more heated than when secondary emission from the plate is absent. Plate emission acts to reduce the apparent plate loss $P_p$ so that the plate is heated more than $P_p$ indicates. Secondary or primary emission from the plate is undesirable and the effects are most noticeable when the tube is operated in the region of high efficiency and large power output, which occurs when the plate-load resistance is medium or high in value. The calculated operating characteristics and measured charts by the 60-cycle method take full account of the effects of secondary emission from the plate.

Primary emission from the grid is undesirable as it reduces the efficiency of operation and increases the driving power. Its effects are not included in the method of calculation from the static characteristic curves but are correctly accounted for in the 60-cycle test method.

It is a pleasure to acknowledge the valuable assistance of Mr. and Mrs. Robert I. Sarbacher and Mr. John Hibbert. Mr. Sarbacher has been an indispensable helper in all phases of this work. He, aided by his wife and Mr. Hibbert, have taken much of the data and carried through the calculations of the charts from the static characteristic curves.

The author, in behalf of the university, wishes to thank the General Electric Company for the gift of a special transformer used in the 60-cycle method, and the Bell Telephone Laboratories and the Raytheon Production Corporation for the loan and gift of vacuum tubes.

## Nomenclature

$\bar{E}_B$, Steady component of plate potential.
$\bar{E}_C$, Steady component of control grid potential.
$(E_p)_{max}$, Maximum value of the alternating component of plate potential.
$(E_g)_{max}$, Maximum value of the alternating components of grid potential.
$(I_p)^1_{max}$, Maximum value of the fundamental component of plate current.
$(I_g)^1_{max}$, Maximum value of the fundamental component of grid current.
$(I_p)_{Av}$, Average value of plate current.
$(I_g)_{Av}$, Average value of grid current.
$e_p$, Instantaneous plate potential.
$e_g$, Instantaneous grid potential.
$i_p$, Instantaneous plate current.
$i_g$, Instantaneous grid current.
$u_p$, Amplification factor.
$(R_b)_\omega$, Resistance of the plate load to the fundamental component of plate current.
$P_b$, Power output, or power dissipated in the plate load.
$P_p$, Power fed to the plate.
$P_d$, Driving power.
$P_g$, Power fed to the grid.
$(Eff)_p$, Plate circuit efficiency.
$(P.A.)$, Power amplification.

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# Cathode Spot Fixation and Mercury Pool Temperatures in an Ignitron

J. Slepian and A. H. Toepfer
*Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pennsylvania*
(Received April 16, 1938)

During development of a metal ignitron tube for high power rectification, some observations were made by thermocouples of the temperatures just below the surface of the mercury pool at a point near the ignitor at the center of the pool, and at a point at the edge of the pool. A section of the tubes tested is shown in Fig. 1, with the positions of the thermocouples indicated. Two such tubes were operated in a full wave rectifier circuit, converting power of 60 cycles into direct current driving a motor. Data as given in Table I were typical and showed that the temperature at the center fluctuated widely under constant conditions of load and water cooling.

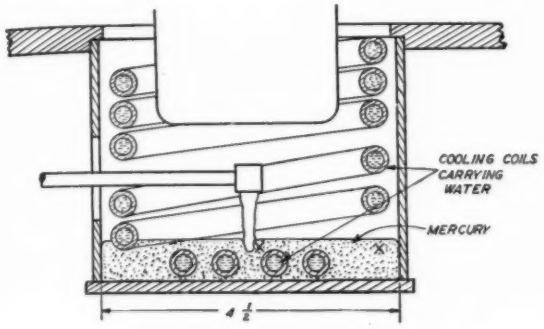


Fig. 1. Ignitron tested. $X$, $X$, position of thermocouples.

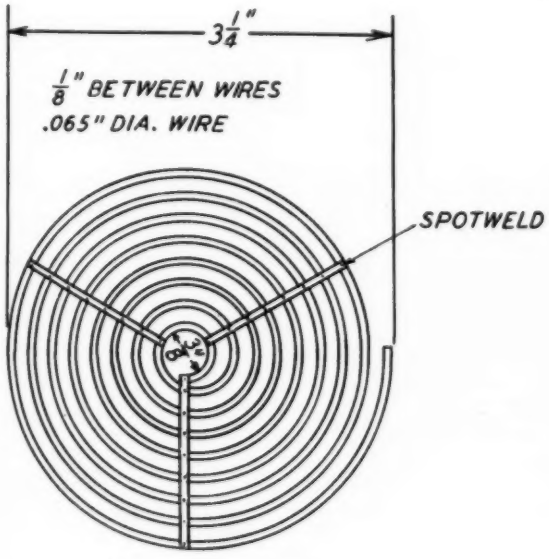


Fig. 2. Floating molybdenum grid spot fixer.

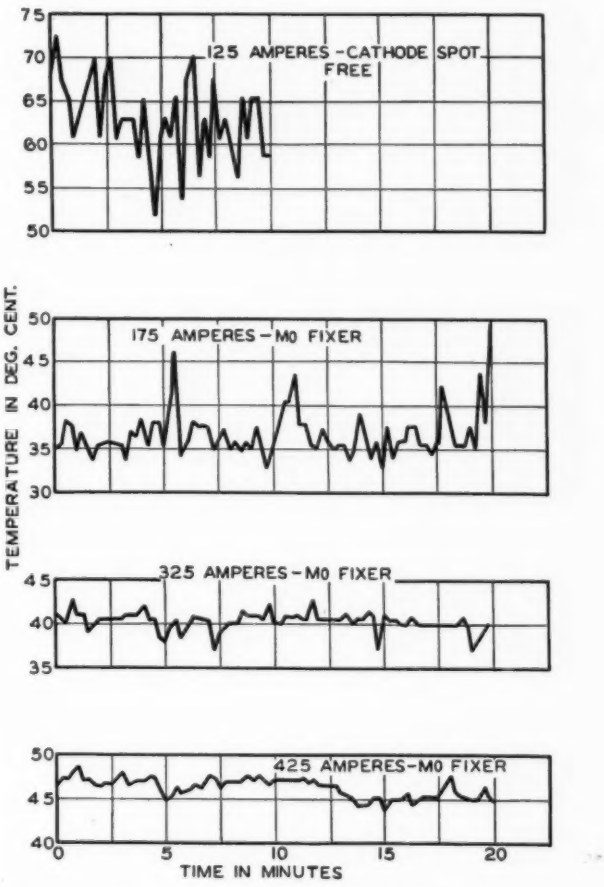
125 AMPERES - CATHODE SPOT FREE
175 AMPERES - MO FIXER
325 AMPERES - MO FIXER
425 AMPERES - MO FIXER
TEMPERATURE IN DEG. CENT.
TIME IN MINUTES

Fig. 3. Temperature of center thermocouple at 15 second intervals.

The magnitudes of the temperature rise are much smaller than are calculated from the quantity of heat observed delivered to the cathode cooling coil, if all this heat is assumed to be transferred through the mercury by thermal conduction alone. We may conclude then that the mercury is stirred considerably by the

TABLE I.

| Date | Time | Amp. d.c. per tube | Volts d.c. | Water Temperature | | Water Rate cc/min. | Thermocouple Temperature | |
|---|---|---|---|---|---|---|---|---|
| | | | | In | Out | | Center | Edge |
| 8–8–35 | 3:45 | 100 | 570 | 28°C | 30°C | 4000 | 35°C | 40°C |
| 8–9–35 | 9:00 | 100 | 580 | 28 | 30 | 4000 | 42 | 41 |
| 8–9–35 | 9:45 | 100 | 580 | 28 | 30 | 4000 | 45 | 41 |
| 8–9–35 | 11:15 | 100 | 570 | 28 | 30 | 4000 | 53 | 41 |
| 8–9–35 | 3:30 | 100 | 570 | 28 | 30 | 4000 | 37 | 42 |

cathode spot of the arc, and that this stirring effect strongly promotes transfer of heat from the surface of the mercury to the cooling coil. We also conclude that this stirring is variable in its intensity under apparently constant load and cooling conditions.

The ignitron tubes were then opened up and spiral molybdenum grids as shown in Fig. 2 were floated upon the mercury. It was expected that the cathode spot of the arc would fix at the molybdenum mercury junction in the well-known manner.[1] After a little preliminary running, the mercury pool temperatures were observed to have been remarkably influenced. The temperatures were greatly lowered, and became very steady. Fig. 3 shows some data taken at 15-second intervals.

The arc drop with the molybdenum grid was practically the same as without the grid. Apparently the molybdenum grid caused the stirring of the mercury to be more intense and constant.

[1] Tonks, Physics 6, No. 9, 1935, p. 294.

## A Note on Certain Guided Waves in Slightly Noncircular Tubes

S. A. SCHELKUNOFF
*Bell Telephone Laboratories, New York, N. Y.*
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Amongst electric waves that can be propagated inside metal tubes of circular cross section there are some with attenuation constants diminishing indefinitely with increasing frequency. The purpose of this note is to discuss what happens to these waves as the tubes are slightly deformed from perfect circularity. The conclusion is that in a deformed tube the attenuation constants of the above-mentioned waves diminish with increasing frequency until a certain frequency is reached, depending upon the degree of departure from perfect circularity, and then they begin to increase indefinitely with further increase in frequency.

ELECTRIC wave propagation inside metal tubes takes place in one or more *transmission modes*. Each particular mode is characterized by a definite amplitude distribution of longitudinal displacement current over any normal cross section of the tube.[1,2] This longitudinal current may be either electric or magnetic, depending upon the type of the wave. The amplitude pattern may be conveniently represented by drawing *equidensity lines*, that is, contour lines joining the points of equal longitudinal current density.

Let the displacement current density inside the tube be proportional to $T(x, y)$ or $T(\rho, \varphi)$, depending upon the particular coordinate system; then

$$T(\rho, \varphi) = \text{constant} \qquad (1)$$

[1] John R. Carson, Sallie P. Mead, S. A. Schelkunoff, "Hyperfrequency Wave Guides—Mathematical Theory," Bell Sys. Tech. J. **15**, 310–333 (1936).
[2] S. A. Schelkunoff, "Transmission Theory of Plane Electromagnetic Waves," Proc. I. R. E. **215**, 1457–1492 (1937).

is the family of equidensity lines. These lines are also the lines of force for those field components which are strictly transverse. Thus in transverse magnetic waves the family of curves (1) represents magnetic lines of force and in transverse electric waves the same curves represent electric lines. Furthermore, in transverse magnetic waves (1) represent electric equipotential lines and in transverse electric waves magnetic equipotential lines.

The boundary conditions at the surface of a perfectly conducting tube can readily be stated with reference to contour lines (1). In transverse magnetic waves the longitudinal electric intensity must vanish at the boundary of the tube. This intensity is proportional to the longitudinal displacement current density and hence to $T$; therefore the boundary of the tube must coincide with the line $T=0$. In transverse electric waves the boundary must be perpendicular to the electric lines of force, that is, to the contour lines (1), except at the points where the electric intensity vanishes.

Electromagnetic laws make it impossible for the longitudinal current density $T$ to be an arbitrary function but prescribe that $T$ be a solution of the following differential equation

$$\partial^2 T/\partial x^2+\partial^2 T/\partial y^2=-\chi^2 T, \quad \text{in Cartesian coordinates,}$$

$$\rho^2\partial^2 T/\partial\rho^2+\rho\partial T/\partial\rho+\partial^2 T/\partial\varphi^2=-\chi^2\rho^2 T, \quad \text{in polar coordinates,} \qquad (2)$$

where $\chi$ is a constant. If $T$ is not a solution of this equation then the pattern over any cross section, except the one over which it is enforced, will vary with time and the patterns over any two cross sections will, in general, be different. The shape and the size of the tube are the factors which restrict the constant $\chi$ to a discrete set of values. A particular transmission mode corresponds to each number of this set. The velocity of a wave traveling in a given transmission mode is determined by $\chi$ and by the frequency of the wave.

We now pass to our problem. The attenuation constant of transverse electric waves in an air-filled metal tube[2] is

$$\alpha=\frac{1}{240}\left(\frac{\mu f}{\pi g}\right)^{\frac{1}{2}}\left[\frac{\int_{(s)}(\partial T/\partial s)^2 ds}{\chi^2\iint_{(S)}T^2 dS}(1-\nu^2)^{\frac{1}{2}}+\frac{\int_{(s)}T^2 ds}{\iint_{(S)}T^2 dS}\nu^2(1-\nu^2)^{-\frac{1}{2}}\right] \text{ nepers per unit length,} \qquad (3)$$

where: $\mu$ is the permeability of the tube in henries per unit length, $g$ the conductivity in mhos per unit length, $f$ the frequency, $\nu$ the ratio of the cut-off frequency to $f$, $S$ the area of the cross section of the tube, $s$ the perimeter of this area and $\partial T/\partial s$ denotes the derivative in a direction tangential to the tube. It will be noted that the first term inside the brackets vanishes at the cut-off and then it steadily increases with the frequency and approaches a constant limit, while the second term is infinite at the cut-off and diminishes indefinitely with increasing frequency.

An exceptional situation arises when the first term vanishes identically and the attenuation constant decreases with increasing frequency. This happens when the tube is *circular* and the density function is

$$T(\rho, \varphi)=J_0(\chi\rho) \qquad (4)$$

since in this case the tangential derivative $\partial T/\partial\varphi$ is evidently zero. Electric lines of force are circles and inasmuch as in the present case the boundary of the tube has no other alternative but to be tangential to an electric line of force, this boundary must pass through that particular line which corresponds to the zero value of the electric intensity. This requires that the boundary of the tube be $\rho=a$ where $J_1(\chi a)=0$. The smallest root of this equation is approximately equal to 3.83; assuming the radius of the tube equal to unity, we have $\chi=3.83$.

The exceptional character of the wave with circular electric lines of force[3] has led to a suspicion[4] that this wave is essentially unstable and

[3] This wave is sometimes called the "$H_0$-wave."

[4] L. Brillouin, "Propagation of Electric Magnetic Waves in a Tube," Rev. Gén. de l'Électricité **40**, 227–239 (1936).

that a slight departure from perfect circularity would destroy the wave by splitting it into two components of *comparable intensities* and traveling with different velocities so that the field pattern will be continuously changing along the tube. Our analysis does not reveal any instability in this sense. In what follows we obtain a wave which resembles the original wave in the perfectly circular tube as closely as we wish and this wave travels with a perfectly definite velocity and maintains its pattern.

It is only when two waves belonging to *distinct* transmission modes *having the same cut-off frequencies* are superposed, that a slight deformation of the tube may upset the configuration. In this case the deformation will usually affect the two transmission modes differently. Such situations can readily occur in square tubes. Thus in a rectangular tube there exists a pair of related transmission modes in which electric lines are parallel to either one or the other pair of sides of the rectangular cross section. The velocity of a wave traveling in either of these modes depends upon the length of those sides to which the electric lines are perpendicular. In a square tube two waves of equal intensities traveling in these two modes can be superposed to form a wave in which central electric lines of force are nearly circular. Such a wave will maintain its pattern as long as the tube remains perfectly square. As soon as the square tube is slightly deformed into a rectangular tube, the original components of the resultant wave will travel with different speeds and the wave pattern will change. This was shown by Brillouin in his paper. However, any conclusion with regard to square tubes does not apply *per se* to tubes of any other cross section. In fact, if we insert into the square tube a perfectly conducting diagonal partition perpendicularly to the lines of electric force, the electrodynamic equilibrium is not disturbed. The partition imposes, however, an electrodynamic constraint on the two component waves and the latter are forced to coexist; consequently, a slight deformation in the shape of the tube will produce only a slight deformation in the wave pattern.

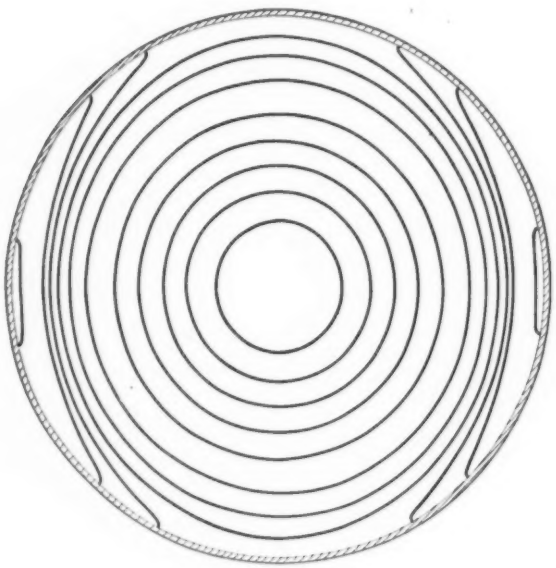

FIG. 1. Electric lines of force for the "$H_0$-wave" in a slightly noncircular tube.

The above discussed question of stability should not be confused with the question of production of new transmission modes when a sudden change takes place in the shape or the size of a given tube. Under these circumstances new waves are always produced regardless of the particular character of the original wave. In contrast with the previously discussed case, the intensities of new waves depend on the "size" of the discontinuity and are small for small discontinuities. Some of these new waves are rapidly attenuated and can be ignored at a distance from the discontinuity.

As the circular tube is deformed, the properties of the wave with circular lines of electric force (the "$H_0$-wave") change gradually and not suddenly. This is true of the attenuation constant as well as of the pattern formed by the electric lines of force. Instead of decreasing indefinitely, the attenuation constant will diminish with increasing frequency only until a certain frequency is reached, depending upon the degree of departure from perfect circularity, and then it begins to increase indefinitely with a further increase in frequency. By making the tube more nearly circular, the turning point can be placed as far in the frequency range as we desire.

We now pass to the analytical details. As the tube is squeezed at two diametrically opposite points, the metal will come in contact with electric lines of non-zero intensity. These lines must break and bend so as to become perpendicular to

the boundary of the tube. Thus a new picture of the lines of force will resemble that shown in Fig. 1.

In order to arrive at this fact analytically, we shall follow the line of the least resistance, by slightly deforming the field and choosing the shape of the tube to fit it rather than chosing the shape of the deformed tube and then calculating the proper field. The second course could be carried through but it presents no advantage in our case. Thus we shall consider a density function which differs but slightly from (4)

$$T(\rho, \varphi) = J_0(\chi\rho) + uJ_2(\chi\rho) \cos 2\varphi, \qquad (5)$$

where $\chi$ is a root of $J_1(x) = 0$. The coefficient $u$ is taken small compared with unity. Fig. 2 represents electric lines for $u = 0.1$ and $\chi = 3.83$. As we have anticipated some circular electric lines have become broken and joined with the neighboring lines to form crescent-like contours. The deformed boundary of the tube must be perpendicular to these lines. The smaller is $u$, the narrower is the region of crescent-like lines of force and the less appreciable is the deformation of the tube.

The differential $dT$ along a level line vanishes; thus along such lines

$$\chi[-J_1(\chi\rho) + uJ_2'(\chi\rho) \cos 2\varphi]d\rho - 2uJ_2(\chi\rho) \sin 2\varphi d\varphi = 0. \qquad (6)$$

From this we have

$$\frac{d\varphi}{d\rho} = \frac{\chi[-J_1(\chi\rho) + uJ_2'(\chi\rho) \cos 2\varphi]}{2uJ_2(\chi\rho) \sin 2\varphi}. \qquad (7)$$

When $u = 0$, $d\varphi/d\rho$ is infinite; this means that the radius drawn from the origin to a point on a particular level line is independent of the angular coordinate and that level lines are circles coaxial with the origin. When $u$ is merely very small compared with unity, $d\varphi/d\rho$ is *usually* very large and consequently level lines approximate circles. The exceptional conditions will be encountered in the region near the circle $\rho = 1$ where $J_1(\chi)$ is small so that the numerator can be also small and can even vanish. The vanishing of $d\varphi/d\rho$ marks the points at which the level lines are *tangential* to the radii. The locus of these points is given by

$$-J_1(\chi\rho) + uJ_2'(\chi\rho) \cos 2\varphi = 0. \qquad (8)$$

Since $u$ is very small and $\cos 2\varphi$ is never greater than unity, this locus must lie in the neighborhood of $\rho = 1$ where the numerator in (8) vanishes.[5] Let $\rho = 1 + \delta$, where $\delta$ is a small quantity; then we have approximately

$$J_1(\chi\rho) = J_1(\chi + \chi\delta) = \chi J_1'(\chi)\delta = \chi J_0(\chi)\delta,$$
$$J_2'(\chi\rho) = J_2'(\chi) = -0.5\, J_3(\chi). \qquad (9)$$

Substituting in (8), we have

$$\delta = -\frac{uJ_3(\chi)}{2\chi J_0(\chi)} \cos 2\varphi = 0.136u \cos 2\varphi. \qquad (10)$$

The final answer is calculated for the special case $\chi = 3.83$. Evidently the locus (8) crosses the unit circle on the straight lines bisecting the coordinate axes and it represents an oval which is a circle slightly flattened at the top and at the bottom. Since the tube is perpendicular to electric lines, its cross section will approximately coincide with the curve (8).

In order to calculate the attenuation constant from (3), we have to find the derivatives of $T$ along the curve (8); these are approximately

$$\partial T/\partial\varphi = -2uJ_2(\chi) \sin 2\varphi, \quad \partial T/\partial\rho = 0. \qquad (11)$$

Thus at the surface of the tube $\partial T/\partial s$ is proportional to $u$, except for higher powers of the small

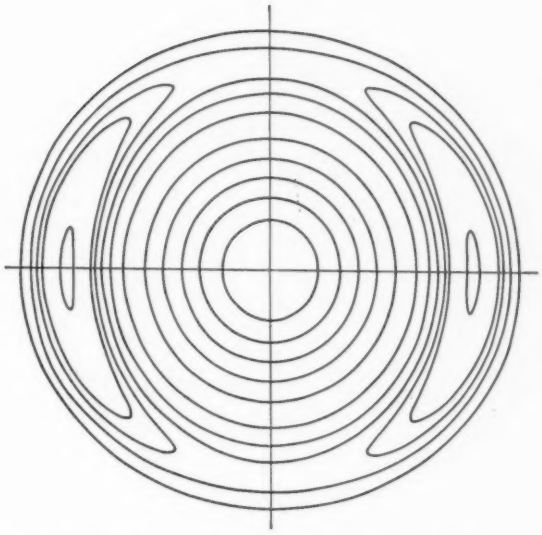

FIG. 2. Electric lines of force for a wave which is deformed from that with circular lines of force.

[5] Of course, there will be other loci in the neighborhoods of circles determined by $\chi\rho$ = a zero of $J_1(x)$. We are interested only in the locus approximating the original cross section of the tube.

quantity $u$, and consequently the first term in the attenuation constant (3) is nearly proportional to $u^2$ and rapidly diminishes as the tube approaches perfect circularity. For sufficiently small departures from perfect circularity, the attenuation constant of nearly circular tubes is dominated by the second term in the region from the cut-off to some frequency which increases indefinitely as the tube becomes more nearly circular. In this range, which can be made as wide as we please the attenuation constant diminishes with increasing frequency. For circular tubes the range becomes infinitely wide.

It is evident from (10) that the largest radius corresponds to $\varphi=0$ and the smallest to $\varphi=90°$; in the former case the radius is $1+0.136u$ and in the latter $1-0.136u$. The ratio of the largest diameter of the tube to the smallest exceeds unity approximately by an amount

$$\Delta=0.272u. \tag{12}$$

This quantity $\Delta$ may be chosen to represent the degree of departure of the tube from perfect circularity.

Making the necessary substitutions in (3), we obtain (after somewhat lengthy but perfectly straightforward calculations) the following approximate formula for the attenuation constant in a deformed circular tube

$$\alpha=1/120(\mu f/\pi g)^{\frac{1}{2}}[\nu^2(1-\nu^2)^{-\frac{1}{2}}+p(1-\nu^2)^{\frac{1}{2}}], \tag{13}$$

where[6]

$$p=2[J_2(3.83)/J_3(3.83)]^2\Delta^2=1.84\,\Delta^2. \tag{14}$$

Minimizing $\alpha$ with respect to the frequency we find that the attenuation constant reaches its minimum when

$$\nu=\left(\frac{p}{3}\right)^{\frac{1}{2}}=\frac{(2)^{\frac{1}{2}}J_2(3.83)}{(3)^{\frac{1}{2}}J_3(3.83)}\Delta=0.783\Delta. \tag{15}$$

Thus if the largest diameter exceeds the smallest by 1 percent, the minimum occurs at a frequency which is 128 times as great as the cut-off.

The foregoing method of studying wave propagation in slightly noncircular tubes can be applied with equal facility to all other transmission modes and the deformations of the original tube can be made more complex by superposing several amplitude distributions corresponding to the *same* value of $\chi$ and then imposing a proper electrodynamic constraint in the form of a perfectly conducting tube.

[6] The coefficient of the first term in the brackets is also affected by the departure from perfect circularity; a term proportional to $\Delta^2$ is added to unity. However, it is the second term that claims our interest.